YMCA UNIVERSITY OF SCIENCE AND TECHNOLOGY,FARIDABAD

M.Sc.(Mathematics) I Semester (Under CBS)(May-2018)

Real Analysis (MTH-501)

M.Marks:60 **Time: 3 hours**

Note: Question paper has two parts, **Part-I and Part-II**. All questions in **Part-I** are compulsory. **Part-II** has six questions out of which four questions have to be attempted by the students.

Part-I

Note: All Questions are compulsory (word limit 20-40 only)

Que.1(a)State Cauchy's criterion for uniform convergence of sequence of functions.

(b)State Weierstrass approximation theorem.

(c)Define Norm of partition and refinement of partition.

(d)State necessary condition for a function to be R-S integrable.

(e)Find the radius of convergence of the given series: $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$.

(f)If $\overline{\lim}|a_n|^{1/n} = \frac{1}{R}$, then prove that the series $\sum a_n x^n$ is convergent (absolutely) for |x|< R and divergent for |x|>R.

(g)If $f(x,y) = 2x^2 - xy + 2y^2$,then find ∂f/∂x and ∂f/∂y at the point (1,2).

(h)State inverse function theorem.

(i)Define Borel sets with examples.

(j)State classical lebesgue dominated convergence theorem. (2 x 10=20)

Part-II

Que.2(a) State and prove Dirichelt's test for uniform convergence of series of functions. (5)

(b)Let $<f_n>$ be a sequence of real valued function defined on the closed interval [a,b] and let

$f_n \in R[a,b]$,for $n \in N$. If $<f_n>$ converges uniformly to the function f on [a,b].

Then prove that

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$$f \in R[a,b] \text{ and } \int_a^b f(x)dx = \lim \int_a^b f_n(x)dx \quad . \qquad (5)$$

MTH501-4

Que.3(a)State and prove mean value theorem for integral calculus. (5)

(b)If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$,then prove that $f \in R(\alpha_1+\alpha_2)$ and

$$\int_a^b f d(\alpha_1+\alpha_2) = \int_a^b f d\alpha_1 + \int_a^b f d\alpha_2 \quad . \qquad (5)$$

Que.4(a)State and prove uniqueness theorem for power series. (5)

(b)If a power series $\sum a_n x^n$ converges for $|x| < R$ and if we define a function $f(x) = \sum a_n x^n$, $|x| <R$, then prove that $\sum a_n x^n$ converges uniformly on$[-R+\in ,R-\in]$, no matter which $\in > 0$ is choosen and that the function f is continuous and differentiable on (-R,R) and $f'(x) = \sum n a_n x^{n-1}$,$|x|< R$. (5)

Que.5(a)Check the continuity of the function at the origin:

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases} \quad . \qquad (5)$$

(b)Show that the function $f(x,y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at (0,0) . (5)

Que.6(a)If A_1 and A_2 are measurable subsets of [a,b] then prove that both $A_1 \cup A_2$ and $A_1 \cap A_2$ are measurable. (5)

(b)Prove that the function f on [a,b] is measurable iff one of the following conditions hold:

(i) $\{x : f(x) > \alpha\}$ is measurable set for every real α .

(ii) $\{x : f(x) \geq \alpha\}$ is measurable set for every real α . (5)

Que.7(a)Expand $x^2y + 3y - 2$ in powers of (x-1) and (y+2) using Taylor's expansion. (5)

(b)If P* is a refinement of P, then prove that $L(P,f,\alpha) \leq L(P^*,f,\alpha)$ and

$U(P^*,f,\alpha) \leq U(P,f,\alpha)$ (5)