YMCA UNIVERSITY OF SCIENCE & TECHNOLOGY, FARIDABAD

M.Sc. Mathematics ,Ist semester

Differential Equations (MTH-505)

Time: 3 Hours **Max. Marks: 60**

Instructions:
1. *It is compulsory to answer all the questions (2 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART –A

Q1 (a) Solve $(y \cos x + 1)\,dx + \sin x\, dy = 0$. (2)

(b) Solve $(D^2 - 4D + 3)y = e^x \cos 2x$. (2)

(c) Find the characteristic values of $\frac{dx}{dt} = 6x - 3y, \frac{dy}{dt} = 2x + y$. (2)

(d) Find adjoint equation of $t^2 \frac{d^2x}{dt^2} + 7t\frac{dx}{dt} + 8x = 0$. (2)

(e) When do we call a critical point a saddle point.

(f) Give an example of non linear differential equation.

(g) State Sturm separation theorem.

(h) Define the term Wronskian.How is it used to check linear dependence of vector functions.

(i) Define Lipchitz condition w.r.t. y.

(j) Prove that $P_n(1)=1$.

PART B

Q1 (a) Solve by Frobeniusmethod , $2x^2y''+xy'+(x^2-3)y = 0$ in $0 < x < R$

(b) Find series solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$ *in powers of* x *about the point* x_0. [5+5]

Q2(a) State and prove Sturm comparison theorem.

(b) Find eigen values and eigen functions of Sturm Liouville problem $X''+\lambda X = 0, \quad X'(0) = 0, X'(L) = 0$. [5+5]

Q3 (a)Consider the system by Liapunov's direct method $\frac{dx}{dt} = -x + y^2$, $\frac{dy}{dt} = -y + x^2$ *and* E *defined by* $E(x,y) = x^2 + y^2$.

(b) Find the first three approximations of $\frac{dy}{dx} = x + y$, $y(0) = 1$. [5+5]

Q4(a) State and prove the existence and uniqueness theorem for first order equation. [10]

Q5(a) Find the nature of critical point (0,0)of the system

$$\frac{dx_2}{dt} = 3x_1 - 8x_2$$

$\frac{dx_1}{dt} = 2x_1 - 7x_2$, determine whether the point is stable. [5]

Q5(b) Find series solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0$

Q6(a)Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$.

Q6 (b) Find the general solution of the differential equation

$$\frac{dx_1}{dt} = 5x_1 - x_2$$

$\frac{dx_2}{dt} = 3x_1 + x_2$ [5+5]