Roll No. ....

Total Pages : 4

240102

December, 2019

M.Sc. (Mathematics) - Ist SEMESTER

ABSTRACT ALGEBRA (MATH 17-102)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Define normal and composition series with an example. (1.5)

(b) Explain permutation group with example. (1.5)

(c) If p is a prime number and G is a non-abelian group order p^3, then prove that the center Z of G has exactly p elements. (1.5)

(d) Find all non-isomorphic abelian group of order 16. (1.5)

(e) If $o(G) = p^n$ (p is prime and $n > 1$) and if H is a subgroup of G of order p^{n-1}, then $H < G$. (1.5)

(f) Let R denote the set of all real-valued continuous functions on [0, 1]. Let $f, g \in R$. Define

$$(f+g)(x) = f(x) + g(x),$$

$$(f.g)(x) = f(x).g(x), \quad \forall x \in [0, 1].$$

Then, R is a commutative ring with unity. (1.5)

(g) Show that the intersection of two prime ideals of a ring R may not be a prime ideal of R. (1.5)

(h) Prove that every field is a P.I.D. (Principal Ideal Domain). (1.5)

(i) Show that $[Q(\sqrt{2}) : Q] = 2$. (1.5)

(j) Discuss the irreducibility of $f(x) = x^4 + 1$, over rationals. (1.5)

PART - B

2. (a) Let G be a group and $N \triangleleft G$ such that both N and G/N are solvable, then G is also solvable. (7)

(b) Prove that a group G is solvable iff $G^k = \{e\}$, for some $k \geq 1$. (8)

2. (a) Prove that any two composition series of a finite group G are isomorphic. (8)

(b) Prove that any two sylow p-subgroups of a finite group G are conjugate. (7)

4. (a) If p is a prime number and p^m divides $o(G)$, then G has a subgroup of order p^m. (5)

(b) If G is the internal direct product of its subgroups $H_1, H_2, ..., H_n$, then $G \approx H_1 \times H_2 \times ... \times H_n$. (10)

5. (a) Prove that any homomorphism of a field is either an isomorphism or takes each element into 0. (7)

(b) Show that $< x+2 >$ is a maximal ideal of $Q[x]$ and hence $\frac{Q[x]}{< x+2 >}$ is a field. (8)

6. (a) Prove or disprove that $Z[\sqrt{-5}]$ is a U.F.D. (Unique Factorization Domain). (7)

(b) Find the degree of the splitting field of $x^3 - 2$ over Q. (8)

7. (a) Every finite extension of a field F is an algebraic extension. However, the converse is not true. (8)

(b) Construct a field having 121 elements. (7)
