Roll No. ....

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240103

December, 2019

M.Sc. (Mathematics) - I SEMESTER

Ordinary Differential Equations (MATH17-103)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Define equicontinuous family of functions. (1.5)

(b) Find the general solution of

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=0.$$ (1.5)

(c) Write two disadvantages of Euler's method. (1.5)

(d) Find the adjoint equation of

$$(2t+1)\frac{d^2x}{dt^2}+t^3\frac{dx}{dt}+x=0.$$ (1.5)

(e) Solve

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}.$$ (1.5)

(f) Define critical point of a linear autonomous system. (1.5)

(g) Find the curves passing through (0, 1) and satisfying $\sin\left(\frac{dy}{dx}\right)=c$, where cis a constant. (1.5)

(h) Write Lipschitz condition with respect to y. (1.5)

(i) Give an example of a 3rd order, linear, non-homogeneous differential equation. (1.5)

(j) Define Wronskian for n real functions $f_1\, f_2, \ldots, f_n$. (1.5)

PART - B

2. (a) Use Picard's method to find three successive approximations of the solution of the differential equation

$$\frac{dy}{dx}=y-x, \qquad y(0) = 2,$$

Show that the iterative solution approaches the exact solution. (10)

(b) Show that the following initial value problem has a unique solution and hence find the solution

$$\frac{dy}{dx} = 3x^2, \quad y(0) = 0.$$ (5)

3. (a) State and prove Cauchy-Peano existence theorem. (8)

(b) Find the power series solution of the differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$$

in powers of x. (7)

4. (a) State and prove Sturm separation theorem. (7)

(b) Find the characteristic value and the characteristic function of the Sturm-Liouville problem

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x}y = 0, \quad y(1) = 0, \quad y(e^{\pi}) = 0.$$ (8)

5. (a) Determine the nature of the critical point (0, 0) of the following nonlinear system

$$\frac{dx}{dt} = x + 4y - x^2,$$

$$\frac{dy}{dt} = 6x - y + 2xy$$

Also, determine the stability of the critical point. (8)

(b) Verify that the function E defined by $E(x, y) = x^2 + y^2$, is a Liapunov function for the following system

$$\frac{dx}{dt} = -x + y^2,$$

$$\frac{dy}{dt} = -y + x^2. \qquad (7)$$

6. (a) Find the general solution of the linear system

$$\frac{dx}{dt} = 6x - 3y,$$

$$\frac{dy}{dt} = 2x + y. \qquad (7)$$

(b) Consider the linear system

$$\frac{dx}{dt} = 3x + 4y,$$

$$\frac{dy}{dt} = 2x + y.$$

Show that $x = 2e^{5t}$, $y = e^{5t}$ and $x = e^{-t}$, $y = -e^{-t}$ are solutions of the above system. Also prove that these two solutions are linearly independent on every interval $a \leq t \leq b$, and write the general solution of the system. (8)

7. (a) Find the series solution of

$$x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} - (x^2 + 5/4)y = 0,$$

about $x = 0$, by Frobenius method. (8)

(b) Solve $\cos\theta \, dr + (r \sin\theta - \cos^2\theta) d\theta = 0$. (7)