## 240303

## December, 2019 <br> M.Sc. (Mathematics) 3rd SEMESTER Partial Differential Equation (MATH17-115)

Time : 3 Hours]
[Max. Marks : 75

Instructions :

1. It is compulsory to answer all the questions ( 1.5 marks each) of Part-A in short.
2. Answer any four questions from Part-B in detail.
3. Different sub-parts of a question are to be attempted adjacent to each other.

## PART-A

1. (a) Using method of separation of variable, derive an expression for one dimensional heat equation.
(b) Write the expression for heat equation in semi-infinite region.
(c) State the solution of 2-D wave equation in Cartesian coordinates.
(d) Write different types of partial differential equations with examples.
(e) Define Laplace equation and Harmonic function with one property each.
(f) Find symmetry of Green's function in U and on surface of $U$.
(g) Define Heat equation. Also state fundamental solution of Heat equation.
(h) State Reflection Method.
(i) Explain in brief 1-D non-homogeneous wave equation.
(j) State energy method; uniqueness solution of wave equation.

## PART-B

2. (a) Derive an expression for 3-dimensional Laplace equation in Cylindrical coordinates.
(b) Derive an expression for 3-dimensional wave equation in spherical coordinates.
3. (a) Derive an expression for transport equation which shows that if $u$ is known at any point on a line, then it is known everywhere in $\mathrm{R}^{\mathrm{n}} \mathrm{X}(0, \infty)$.
(b) Prove that the value of $u$ at any $x \in U$ is equal to the average value of $u$ over entire ball or over surface of ball centered at $x$ and radius is taken such that ball is contained in U .
4. Find the solution of poisson equation with boundary condition in terms of Green's Function.
5. (a) State and Prove D. Alembert's formula.
(b) State and Prove Euler - Poisson Darboux Equation.
6. (a) Derive the resultant equation in Cartesian coordinates for 3-D Laplace equation.
(b) State and Prove non-homogeneous Transport equation.
7. (a) State and Prove the Dirichlet Principle.
(b) Find the solution of 3-D non-homogeneous wave equation.
