Roll No. ....

Total Pages : 4

240301

December, 2019
M.Sc. (Mathematics) - III SEMESTER
Topology (MATH17-113)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Determine the closures of the following subsets of the ordered square :

$A=\{(1/n)\times 0 : n\in Z_+\}$

$B=\{(1-1/n)\times\frac{1}{2} : n\in Z_+\}$ (1.5)

(b) Find the boundary and the interior of each of the following subsets of R^2

(i) $A = \{x \times y : y = 0\}$.

(ii) $B = \{x \times y : x > 0 \text{ and } y \neq 0\}$. (1.5)

(c) Define Hausdorff Space. (1.5)

(d) Show that the function $f : R \to R$ given by $f(x) = 3x + 1$ is a homeomorphism. (1.5)

(e) Define limit point compact space. (1.5)

(f) State Czech Theorem and Tychonoff's Theorem. (1.5)

(g) Prove that for functions $f : R \to R$ the $\epsilon - \delta$ definition of continuity implies the open set definition. (1.5)

(h) Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on X. If $\mathcal{T}' \supset \mathcal{T}$, what does connectedness of X in one topology imply about connectedness in the other? (1.5)

(i) State Urysohn lemma. (1.5)

(j) State Tietze Extension theorem. (1.5)

PART - B

2. (a) Prove that if $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then the collection

$D = \{B \times C : B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$

is a basis for the topology of $X \times Y$. (8)

(b) Prove that if Y be a subspace of X, A be a subset of Y and $\bar{A}$ denote the closure of A in X, then the closure of A in Y equals $\bar{A} \cap Y$. (7)

3. (a) Prove that if $P : X \to Y$ be a quotient map, Z be a space and $g : X \to Z$ be a map that is constant on each set $P^{-1}(\{y\})$ for $y \in Y$, then g induces a map $f : Y \to Z$ such that $f \circ p = g$. The induced map f is continuous if and only if g is continuous. (8)

(b) Prove that the image of connected space under a continuous map is connected. (7)

4. (a) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X. (8)

(b) Prove that if X has a countable basis then every open covering of X contains a countable subcollection covering X. (7)

5. (a) Prove that Compactness implies limit point compactness, but not conversely. (8)

(b) Prove that every compact subset of a Hausdorff space is closed. (7)

6. (a) Prove that the product of finitely many compact spaces is compact. (8)

(b) Prove that every compact Hausdorff space is normal. (7)

7. (a) Let X be a topological space. Let one-point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighbourhood U of x, there is neighbourhood V of x such that $\overline{V} \subset U$. (8)

(b) Prove that a subspace of a regular space is regular. (7)
