Sr. No.752105

March 2022

M.Sc. PhysicsSemester-I (Reappear)

Mathematical Physics (PHL-101)

Time: 90 Minutes

Max. Marks:25

Instructions:
1. *It is compulsory to answer all the questions (1marks each) of Part -A in short.*
2. *Answer* ***any three*** *questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Find the roots of $z^4 + 1 = 0$. (1)

(b) Separate ln(x+iy) into its real and imaginary parts. (1)

(c) Show that the function $U = x^2 - y^2$ is harmonic. (1)

(d) Evaluate $\int_C \frac{dz}{z(z+\pi i)} dz$ where C is a circle $|z+3i|=1$, by Cauchy's integral formula. (1)

(e) Prove that $P_n(1) = 1$. (1)

(f) Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. (1)

(g) Show that the diagonal elements of a skew-Hermitian matrix are zero or purely imaginary. (1)

(h) Show that eigen values of a unitary matrix are unimodular. (1)

(i) If $F\{f(t)\} = F(s)$ then show that $F\{f(x-a)\} = e^{isx}F(s)$. (1)

(j) If $L\{f(t)\} = L(s)$ then show that $L\{f(at)\} = \frac{1}{a}L\left(\frac{s}{a}\right)$. (1)

PART -B

Q2 (a) Obtain CR conditions in polar form for an analytic complex functions. (3)

(b) Find the residue of the function $f(z) = \frac{z^2}{(z+1)^2(z-2)}$ at its double pole. (2)

Q3 Expand $f(z) = \frac{7z-2}{z^3 - z^2 - 2z}$ in a Laurent series valid for $|z+1| > 3$ (5)

Q4 For Bessel's function of order n, show that $\int_0^1 x J_n(\alpha x).J_n(\beta x)\, dx = 0$, where α and β are the roots of $J_n(x) = 0$. (5)

Q5 Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$. Obtain a matrix *P* which diagonalizes the matrixA and also verify $P^{-1}AP = D$ where *D* is the diagonal matrix. (5)

Q5 (a) Show that the matrix $A = \begin{bmatrix} \frac{\sqrt{2}}{2} & -i\frac{\sqrt{2}}{2} & 0 \\ i\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a unitary matrix. (3)

(b) Prove that the eigen value of a skew-Hermitian matrix is either zero or purely imaginary. (2)

Q6 (a) Find the Fourier transform of the function $f(x) = \begin{cases} 1 \; for \; |x| < a \\ 0 \; for \; |x| > a \end{cases}$ (2)

(b) Evaluate the integral $\int_0^\infty t^3 e^{-t} \sin t \, dt$ using Laplace transformation. (3)

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