

- Instructions:**
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
 2. Answer any four questions from Part -B in detail.
 3. Different sub-parts of a question are to be attempted adjacent to each other.

PART -A

- Q1 (a) What is meant by degrees of freedom? What is the number of degrees of freedom of a body which is constrained to move along a space curve? (1.5)
- (b) State and explain D' Alembert's principle. (1.5)
- (c) Explain holonomic and non-holonomic constraints, giving examples. (1.5)
- (d) Prove that a co-ordinate which is cyclic in the Lagrangian is also cyclic in Hamiltonian. (1.5)
- (e) Define Poisson's brackets and write its fundamental properties. (1.5)
- (f) Explain stable, unstable and neutral equilibria on the basis of potential function. (1.5)
- (g) What is the central force? Are all central forces conservative? (1.5)
- (h) State and explain the Hamilton-Jacobi equation for Hamilton's principle function. (1.5)
- (i) What is chaos? Give an example. (1.5)
- (j) Write the principle of Least action. (1.5)

PART -B

- Q2 (a) Obtain the Hamiltonian H and the Hamilton's equations of motion of a simple pendulum. Prove that H represents the constant of motion and total energy. (10)
- (b) Obtain Lagrangian L from Hamiltonian H and show that it satisfies Lagrange's equations of motion. (5)
- Q3 (a) Show that following transformation is canonical: (5)
- $$Q = \sqrt{2q} e^{\alpha} \cos p \quad P = \sqrt{2q} e^{-\alpha} \sin p \quad \alpha \text{ is a constant}$$
- (b) Apply Hamilton-Jacobi theory to solve the motion of one-dimensional harmonic oscillator. (10)
- Q4 Determine the frequencies and normal modes of vibration of a system of linear triatomic molecule. (15)
- Q5 (a) Explain the following with the local phase flow curves: (5)

(i) stable node (ii) unstable node (iii) hyperbolic point (iv) unstable spiral point (v) elliptic point.

(b) Draw the phase curve of a simple pendulum and match it with the curve (10) representing the potential.

Q6 (a) Find Poisson bracket of $[L_x, L_y]$ where L_x and L_y are angular momentum components. (5)

(b) Define the Hamiltonian and hence derive the Hamilton's canonical equation of motion. (5)

(c) A rigid body capable of oscillating in a vertical plane about a fixed horizontal axis is called compound pendulum. (5)

(i) Set up its Lagrangian (ii) obtain its equation of motion (iii) find the period of the pendulum.

Q7 Write short notes on the following:

(5x3)

(a) Liouville's theorem and its applications

(b) Henon-Heiles Hamiltonian

(c) Damped driven pendulum
