Examination Roll No. ....

YMCA UNIVERSITY OF SCIENCE & TECHNOLOGY, FARIDABAD

M.Sc. Physics, Sem – I

MATHEMATICAL PHYSICS (PHL-101)

Time: 3 Hours

Max. Marks:75

Note:
1. ***All** the questions of Part-A are **compulsory**.*
2. *Answer **any four** questions from Part -B in detail.*
3. *Different parts of the same question are to be attempted adjacent to each other.*
4. *Assume suitable standard data wherever required, if not given.*

Part-A

Q.1 (a) Separate $log(x + iy)$ into real and imaginary parts. (1.5)

(b) Obtain C-R equations in Polar form (1.5)

(c) Evaluate $\int_C \frac{e^z}{(z-1)(z-4)} dz$ where C is a circle $|z|=2$, by using Cauchy's Integral formula. (1.5)

(d) Define Jordan's Lemma Theorem. (1.5)

(e) Obtain the roots of the indicial equation corresponding to the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2) y = 0$$ (1.5)

(f) Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (1.5)

(g) Show that every diagonal element of a Skew-Hermitian matrix is either zero of purely imaginary. (1.5)

(h) If a Group has four elements, show that it must be abelian. (1.5)

(i) Evaluate $\int_0^\infty t^2 e^{3t} \sin^2 t \, dt$ (1.5)

(j) If $F\{f(x)\} = F(s)$ then show that $F\{f(ax)\} = \frac{1}{a} F(\frac{s}{a})$ (1.5)

Part-B

Q.2 (a) Show that $\ln z = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} + ----$ (4)

(b) Expand $f(z) = \frac{7z-2}{z^3 - z^2 - 2z}$ in a Laurent series in the region $|z+1| > 3$. (5)

(c) Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in a Laurent series valid for

(i) $1 < |z| < 3$ (ii) $|z| < 1$ (6)

Q. 3 (a) Using Residue theorem, evaluate $\int_C \dfrac{z^2\, dz}{(z-1)^2 (z+2)}$ where C is $|z| = 3$. (5)

(b) Evaluate the real integral $\int_0^\infty \dfrac{dx}{1+x^6}$ (10)

Q.4 (a) If $F\{f(x)\} = F(s)$ then show that $F\{f^n(x)\} = (-is)^n F(s)$ where $f^n(x)$ is the n^{th} derivative of the function $f(x)$. (5)

(b) Prove that $L\left\{\int_0^t f_1(x) f_2(x)\, dx\right\} = F_1(s).F_2(s)$ (5)

(c) Evaluate $L^{-1}\left\{\dfrac{s+4}{s(s-1)(s^2+4)}\right\}$. (5)

Q.5 (a) Prove that $\cos(x \sin\theta) = J_0 + 2J_2 \cos 2\theta + 4J_4 \cos 2\theta + \ldots\ldots..$ (4)

(b) Prove that $xJ_n' = nJ_n - xJ_{n+1}$ (4)

(c) Prove the orthogonal property of Hermite polynomial (7)

Q.6 (a) Prove that $P_n(x)$ is the coefficient of z^n in the expansion of $(1 - 2xz + z^2)^{-1/2}$ in ascending powers of x. (10)

(b) Prove that $nP_n = xP_n' - P_{n-1}'$. (5)

Q.7 (a) Prove that if G be an infinite cyclic group then G has exactly two generators. (4)

(b) Prove that the characteristic roots of a Hermitian Matrix are all real. (4)

(c) Find the eigen values and all the eigen vectors of the matrix A given by

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ (7)