Sr. No. 238101

December, 2019

M.Sc. Physics SEMESTER-I

Mathematical Physics (PHL-101)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

		PART -A	
Q1	(a)	Discuss the singularity of the complex function, $f(z) = \frac{\sin(z-a)}{(z-a)^4}$.	(1.5)
	(b)	Evaluate $\int_C \frac{dz}{z(z+\pi i)} dz$ where C is a circle $\lvert z+3i \rvert = 1$, by Cauchy's integral formula.	(1.5)
	(c)	Find the residue of the function $f(z) = \frac{1}{(z^2+a^2)^2}$ at $z = ia$.	(1.5)
	(d)	Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.	(1.5)
	(e)	Find the value of $\int_{-\infty}^{\infty} e^{-x^2} [Hn(x)]^2 dx$.	(1.5)
	(f)	Prove that $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2-2x}}$.	(1.5)
	(g)	Prove that the group of order three is always cyclic.	(1.5)
	(h)	Show that the characteristic roots of a Hermitian matrix are all real.	(1.5)
	(i)	Evaluate the Fourier Transform of Dirac-delta function.	(1.5)
	(j)	If $L\{f(t)\} = L(s)$ then show that $L\{f(at)\} = \frac{1}{a} L\left(\frac{s}{a}\right)$.	(1.5)
		PART -B	
Q2	(a)	Let f(z)= u(x,y)+iv(x,y) be an analytic function. If u = 3x-2xy, then find the imaginary part of f(z).	(5)
	(b)	Obtain C-R equations in polar form.	(5)
	(c)	Show that $\log z = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} + \ldots\ldots$	(5)
Q3	(a)	By contour integration, prove that $\int_0^{\infty} \frac{\sin mx}{x} dx = \frac{\pi}{2}$.	(10)
	(b)	Expand $f(z) = \frac{z}{z^2-3z+2}$ in a Laurent series in the region $\lvert z-1 \rvert > 1$.	(5)

Q4	(a)	For Legendre's polynomial of order n, prove that $P_n(x)=\frac{1}{2^n\, n!}\cdot\frac{d^n}{dx^n}\left(x^2-1\right)^n$.	(10)
	(b)	Show that $\int_{-1}^{+1}[P_n(x)]^2\, dx=\frac{2}{2n+1}$.	(5)
Q5	(a)	For Bessel's function of order n, show that $\int_0^1 x\, J_n(\alpha x).J_n(\beta x)\, dx=0$, where α and β are the roots of $J_n(x)=0$.	(10)
	(b)	For Hermite polynomial, show that $2n\, H_{n-1}(x)=H'_n(x)$.	(5)
Q6	(a)	Find the Fourier Cosine Transform of $f(x)=5e^{-2x}+2e^{-5x}$.	(5)
	(b)	Find the Laplace transform of $(1+\sin 2t)$.	(5)
	(c)	Evaluate $L^{-1}\left\{\frac{s+4}{s(s-1)(s^2+4)}\right\}$.	(5)
Q7	(a)	Find a matrix P such that $P^{-1}AP$ is diagonal matrix where, $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	(10)
	(b)	Show that three cube roots of unity form an abelian group under multiplication.	(5)

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