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December, 2019
M.Sc. Physics - I Semester
Mathematical Physics (PHL101)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (3 marks each) of Part -A.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (i) Explain the integration round infinite semicircle using Jordan's Lemma. (3)

(ii) Show that

$$\int J_{n+1}(x)\,dx = n\int J_{n-1}(x)\,dx - 2J_n(x) + \text{constant.}$$

(3)

(iii) Show that the covering operations of an equilateral triangle form a group isomorphic to the D_3 group. (3)

(iv) Obtain the Fourier transform of the function f(t) = exp-$|nt|$ (3)

(v) Find the series of sines and cosines of multiples of x which represent f(x) in the interval $-\pi < x < \pi$, where f(x) = 0 when $-\pi < x < 0$; and, f(x) = $(\pi x/4)$ when $0 < x < \pi$. (3)

PART - B

2. (i) Describe evaluation of definite integral in case of integration round a unit circle. Applying the calculus of residue, prove that

$$I = \int_0^{2\pi} \frac{\sin^2 d\theta}{a + b\cos\theta} = \frac{2\pi\left[a - \sqrt{a^2 - b^2}\right]}{b^2} \quad (a > b > 0)$$

(10)

(ii) Evaluate the integral

$$I = \int_0^{\infty} \frac{x^2}{\left[x^2 + 9\right]\left[x^2 + 4\right]^2}\, dx.$$

(5)

3. (i) Define Associated Legendre polynomials and establish the orthogonality relation in these polynomials. (9)

(ii) Prove that $\int_0^{\infty} \frac{x\sin ax}{[a^2 + b^2]}\, dx = \frac{\pi e^{-ab}}{2}$. (6)

4. (i) Derive Rodrigue's relation for Legendre polynomials and use it to prove the orthogonality relation for these polynomials. (10)

(ii) Show that for Bessel function

$$J_0(x) = \frac{2}{\pi}\int_0^1 \frac{\cos xt}{\sqrt{1-t^2}}\, dt. \quad (5)$$

5. (i) Prove that a group of order four may or may not be a cyclic group. Give examples in both the cases. (7)

(ii) Prove that the order of a subgroup of a finite group is a divisor of the order of the group. (4)

(iii) Find the subgroups and corresponding left and right cosets of D_3 group. (4)

6. (i) Find the eigenvalues and corresponding normalised eigenvectors of the matrix

$$B = \begin{bmatrix} 4 & 6 & -2 \\ 6 & 3 & -4 \\ 2 & -2 & 3 \end{bmatrix}$$

and express the digonalised form of this matrix. Verify the results by similarity transformation. (12)

(ii) List essential characteristics of Hermitian and Unitary matrices. (3)

7. (i) Explain method of Fourier series representation and obtain the Fourier series representation for a full wave rectifier. (9)

(ii) Determine the Laplace transform of :

(a) $e^{-pt} \sinh (qt)$.

(b) $5e^{-7t} \cos 4t$. (6)
