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# 300202

**May 2019**

**B.Tech. (Civil)-IInd Semester**

**MATHEMATICS–II**

**(Differential Equations)**

**(BSC106B)**

Time : 3 Hours]

[Max. Marks : 75

*Instructions :*

*(i)* *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*

*(ii)* *Answer any four questions from Part-B in detail.*

*(iii)* *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART–A

**1.** (a) Solve $(y \log y)dx + (x - \log y)dy = 0$. (1.5)

(b) Solve $p = \sin (y - xp)$. (1.5)

(c) Express $f(x) = x^2 + 2x + 1$ in terms of Legendre polynomials. (1.5)

(d) Find the regular points of the differential equation

$$x(1-x)\frac{d^2y}{dx^2}-(1+3x)\frac{dy}{dx}-y=0\cdot \quad (1.5)$$

(e) Form the partial differential equation from the relation $z = f(x + it) + g(x - it)$. (1.5)

(f) Solve $(pq - p - q)(z - px - qy) = pq$ (Clairut's equation). (1.5)

(g) State Duhamel's principle for one-dimensional wave equation. (1.5)

(h) Solve $(2D^2 + 5DD' + 2D'^2)z = 0$. (1.5)

(i) Obtain the differential equation of the coaxial circles of the system $x^2 + y^2 + 2ax + c^2 = 0$, where '$c$' is a constant and '$a$' is a variable. (1.5)

(j) Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$. (1.5)

## PART–B

2\. (a) Solve $y = 2px + p^n$. (7)

(b) Solve $y = 2px + y^2p^3$ (for $x$). (8)

3\. (a) Solve by Method of variation of parameters :

$(D^2 - 2D + 1)y = e^x \log x$, where $D = d/dx$. (7)

(b) Prove that

(i) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n+1}(x)$



(ii) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)\cdot$ (8)

4\. (a) Solve the non-linear partial differential equation $px + qy = pq$. (7)

(b) Solve the linear partial differential equation

$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. (8)

5\. (a) Solve the wave equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u(x, 0) = 3 \sin n\pi x$, $u(0, t) = 0$ and $u(1, t) = 0$, where $0 < x < 1$, $t > 0$. (7)

(b) Solve $(D^2 - DD')z = \cos x \cos 2y$. (8)

6\. (a) Solve $(1 + xy)ydx + (1 - xy)xdy = 0$. (7)

(b) Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$. (8)

7\. (a) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ by Lagrange's method. (7)

(b) Derive the solution for three-dimensional Laplace's equation in cylindrical form. (8)

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