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323202

May, 2019

M.Tech. - II SEMESTER

Optimal Control Theory (MEI202)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (i) Check whether these vectors are linearly independent or not?

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}.$$

(ii) Define complete state controllability and observability in discrete domain.

(iii) Define Minimal Polynomial. What is its significance?

(iv) State & Prove Principle of Duality.

(v) Define Minimal Polynomial. How is it useful in optimal control?

(vi) What are the various constraints on matrices A, B, H, Q & R in continuous time linear state regulator problem?

(vii) Write the Expression for Hamiltonian fn. while explaining the each term involved.

(viii) What is constrained minimization? Write the expression for augmented g in this case.

(ix) Prove that eigenvalues remain invariant under a linear transformation.

(x) What is state observer? How is it useful in designing a suboptimal feedback controller? (1.5×10=15)

PART–B

2. (i) Find the trajectories in the (t, x) plane which will extremize $J(x)=\int_0^{t_1}(t\dot{x}+\dot{x}^2)dt$ when $t_1 = 1$, $x(0) = 1$, $x(1)$ is free. 7.5

(ii) Consider the following system :

$$\dddot{y}+6\ddot{y}+11\dot{y}+6y=6u$$

Obtain the state space representation of the system in diagonal canonical form. 7.5

3. (i) Find the optimal control $u^*(t)$ for the system $\dot{x}=u$; $x(0) = 1$ which minimizes

$$J=\frac{1}{2}x^2(4)+\frac{1}{2}\int_0^4 u^2 dt.$$ 7.5

(ii) Derive Euler Lagrange equation. 7.5

4. (i) A discrete-time system has state equation given by

$$x(k+1)=\begin{bmatrix}0 & 1\\ -10 & -7\end{bmatrix}x(k).$$

Use Caley-Hamilton approach to find out the state transition matrix. 7.5

(ii) Derive Hamilton – Jacobi equation for the continuous time process. 7.5

5. (i) Define Pontryagin's Minimum Principle. Write all the steps involved in solving an optimal control problem using Minimum Principle. 7.5

(ii) How do you define an optimal control problem? Discuss various types of optimal control problems. 7.5

6. Write short notes on :

(i) Dynamic Programming. 7.5

(ii) Discrete time Linear State Regulator. 7.5

7. (i) Prove that the system defined by

$$\dot{x} = Ax, \quad y = Cx$$

where x = state vector (n vector), y = output vector (m vector), A = $n \times n$ matrix, C = $m \times n$ matrix, $m \leq n$, is completely observable if and only if the composite $mn \times n$ matrix P, where

$$P = [CCA \ldots CA^{n-1}]$$

is of rank n. 7.5

(ii) For the system defined by:

$$x(k+1) = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} u(k)$$

Check the controllability and find the complete control sequence to bring the initial state $x(0) = 0$ to final state x^1 to 0.2. What is the controllability index of the system? 7.5