SOME INVESTIGATIONS INTO NOISE ANALYSIS OF ELECTRONIC CIRCUITS

THESIS

submitted in fulfillment of the requirement of the degree of

DOCTOR OF PHILOSOPHY

to

J.C. BOSE UNIVERSITY OF SCIENCE & TECHNOLOGY, YMCA

by

DUSHYANT KUMAR SHUKLA

Registration No.: YMCAUST/Ph 21/2011

Under the Supervision of

Prof. (Dr.) MUNISH VASHISHATH — **Supervisor**

Dr. TARUN RAWAT — **Co-Supervisor**

Faculty of Engineering and Technology
J.C.BOSE University of Science & Technology, YMCA
Sector-6, Mathura Road, Faridabad, Haryana, India

JULY, 2021

DECLARATION

I hereby declare that this thesis entitled **"SOME INVESTIGATIONS INTO NOISE ANALYSIS OF ELECTRONIC CIRCUITS"** by "**DUSHYANT KUMAR SHUKLA**"**,** being submitted in fulfillment of the requirements for the Degree of Doctor of Philosophy in "ELECTRONICS ENGINEERING" under Faculty of Engineering and Technology of J.C.BOSE University of Science & Technology, YMCA, Faridabad, during the academic year 2020-21, is a bona fide record of my original work carried out under guidance and supervision of **Prof. (Dr.) MUNISH VASHISHATH, Supervisor, Professor, Electronics Engg, J.C.BOSE University of Sc. & Tech., YMCA, Faridabad** and **Dr. TARUN RAWAT, Co-Supervisor, Associate Professor, Electronics & Communication Engg, NSUT, New Delhi** and has not been submitted elsewhere.

I further declare that to the best of my knowledge, the thesis does not contain any part of any work which has been submitted for the award of any degree either in this university or in any other university.

DUSHYANT KUMAR SHUKLA

Registration No.: YMCAUST/Ph 21/2011

CERTIFICATE

This is to certify that this Thesis entitled **"SOME INVESTIGATIONS INTO NOISE ANALYSIS OF ELECTRONIC CIRCUITS"** by "**DUSHYANT KUMAR SHUKLA**"**,** submitted in fulfillment of the requirement for the Degree of Doctor of Philosophy in "ELECTRONICS ENGINEERING" under Faculty of Engineering and Technology of J.C.BOSE University of Science & Technology, YMCA, Faridabad, during the academic year 2020-21, is a bonafide record of work carried out under our guidance and supervision.

We further declare that to the best of our knowledge, the thesis does not contain any part of any work which has been submitted for the award of any degree either in this university or in any other university.

Prof. (Dr.) MUNISH VASHISHATH,
Supervisor, Professor
Electronics Engg,
J.C.BOSE University of Sc. & Tech., YMCA, Faridabad

Dr. TARUN RAWAT
Co-Supervisor, Associate Professor,
Electronics & Communication Engg,
NSUT, New Delhi

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my Supervisor **Prof. (Dr.) Munish Vashishath** and Co-Supervisor **Dr. Tarun Rawat** for giving me the opportunity to work in this area. It would never be possible for me to take this thesis to this level without their innovative ideas and their relentless support and encouragement.

This thesis would not have been possible without the continuous and unconditional support of my family members and friends who gave me strength and will to succeed.

I want to express my sincere thanks to all those who directly or indirectly helped me at various stages of this work.

Above all, I express my indebtedness to the "ALMIGHTY" for all His blessings and kindness.

DUSHYANT KUMAR SHUKLA

Registration No.: YMCAUST/Ph 21/2011

ABSTRACT

In this thesis, noise analysis of different Bipolar Junction Transistor (BJT), Field Effect Transistor (FET) amplifiers and Metal Oxide Semiconductor (MOS) differential amplifier is performed. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. Noise can affect the circuit performance in the large extent and put a limit on the performance of the circuit. It is very important to predict noise performance accurately especially in analog blocks in the mixed signal system design so that overall system will work correctly. In this thesis the effect of external noise on different BJT and FET amplifiers and MOS differential amplifier is analysed. In this thesis the effect of noise is analysed at high frequencies. Circuits are also considered for variable load resistance and capacitive load. In this work the effects of device parameters on the noise performance of MOS differential amplifier is investigated. Then comparison of the noise performance of the BJT and MOS differential amplifier is done.

Electronic circuits can be represented by differential equations, in which all the parameters are deterministic. Such differential equations are called ordinary differential equations. The solution of such equations provides voltage or current of the circuit. The solution of these ordinary differential equations do not include the effect of noise as in these differential equations no source of noise is considered. In order to analyse the effect of noise, noise/random signal source is added in the ordinary differential equation. When noise/random term is added to the ordinary differential equation, it becomes stochastic differential equation. It is assumed that noise is a white Gaussian noise. Although it is an ideal condition, when it is assumed that the noise is white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectral density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for

extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. In this work a time domain method using SDE to analyse the effect of noise on different amplifiers is used. The autocorrelation function and other statistics like mean and variance of the output using stochastic differential equations are obtained. In this work, an approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

TABLE OF CONTENTS

LIST OF FIGURES/GRAPHS

LIST OF ABBREVIATIONS

Symbol	**Description**
SDE	Stochastic Differential Equation
BJT	Bipolar Junction Transistor
FET	Field Effect Transistor
MOS	Metal-Oxide-Semiconductor
CB	Common Base
CC	Common Collector
CE	Common Emitter
CS	Common-Source
CD	Common-Drain
CG	Common-Gate

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Any electric circuit is affected by two types of noises: internal noise and external noise. Internal noise is created within the circuit or device. The examples of internal noise are thermal noise, shot noise, transit-time noise, flickers etc. Thermal noise in the circuit is caused by the discreteness of the electric charges [1-2]. At the finite temperature, every dissipative electrical circuit has thermal noise. Shot noise is caused by random variations in the arrival of electrons or holes at the output electrode of the amplifier device. If the time taken by an electron to travel from the emitter to the collector of a transistor becomes significant to the time period of the signal, random fluctuations take place and create random noise, which is called transit-time noise. The noise which is created outside the circuit is called external noise. The examples of external noise are atmospheric noise, extraterrestrial noise and industrial noise.

Noise can affect the circuit performance in the large extent and put a limit on the performance of the circuit. It is very important to predict noise performance accurately especially in analog blocks in the mixed signal system design so that overall system will work correctly. In the analysis of noise, a common method which is used is to add a noise term to the right side of the deterministic equation. First such model is formulated by Langevin in 1908 to describe the velocity of a particle moving in a random force field.

If deterministic differential equations are used to model any circuit, the effects of both types of noise will be ignored. To include the effects of both external and internal noise, one can replace the input and internal parameters in the deterministic model by random process. Such type of random differential equation may be interpreted as stochastic differential equation (SDE) [3-4]. Solutions of these equations represent Markov diffusion processes, the prototype of which is the Brownian motion process also called Wiener process [5–8].

Generally, noise analysis electric circuit is performed in frequency domain. In this thesis, a time domain method based on solving stochastic differential equation is used. To derive and compute non-Gaussian, non-stationary and nonlinear stochastic characterization of both amplitude and phase noise in an oscillator, the stochastic differential equation approach is adopted in [9]. Using dissipative Hamiltonian systems theory of ordinary differential equations (ODEs) and stochastic differential equations, a qualitative theory of Josephson circuit family is developed in [10]. The stochastic differential equation approach was adopted in [11] from simulation point of view for noise analysis. This method is based on linearization of stochastic differential equation about its simulated deterministic trajectory.

1.2 SDE AND ITS APPLICATIONS

Any electrical circuit which consists of resistor, inductor and capacitor may be modelled by deterministic differential equation in which the coefficient of the deterministic differential equation depends on circuit elements. This equation does not include/ characterize the effects of any noise on the circuit. To include the effect of noise we can add a noise to the input source and to the circuit elements. So when noise is added in the deterministic differential equation, it becomes the stochastic differential equation. It is assumed that the noise source which is added in the circuit is white noise. The noise added in the parameters may be a correlated process. The effect of noise in the electronics circuits is to be observed. Generally the components of electronic circuits are resistors, inductors capacitors, active devices like bipolar junction transistor, field effect transistor, metal oxide semiconductor transistor etc. Electronic circuits can be represented by differential equations, in which all the parameters are deterministic. Such differential equations are called ordinary differential equations. The solution of such equations provides voltage or current of the circuit. But solution of these ordinary differential equations do not include the effect of noise as in these differential equations no source of noise is considered. In order to analyse the effect of noise, noise/random signal source is added in the ordinary differential equation. When noise/random term is added to the ordinary differential equation, it becomes stochastic differential equation.

The ordinary differential equation can be written as

$$\frac{dx(t)}{dt} = f(x(t), t) \qquad t > 0 \qquad (1.1)$$

$$x(0) = x_0$$

The solution of eq. (1.1) is the trajectory as shown in Fig.1.1.

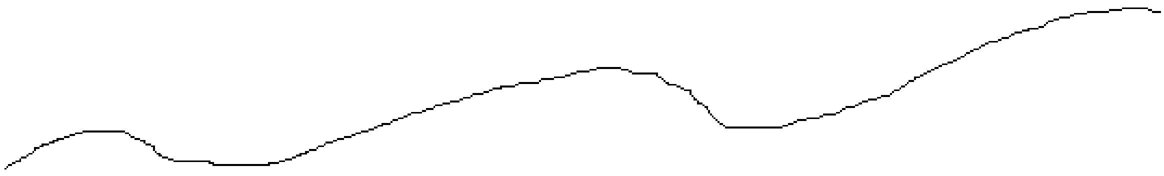

Fig.1.1. Trajectory of an ODE

In many applications, however the experimentally measured trajectories of system modelled by ODE do not in fact behave as predicted. It behaves as shown in Fig.1.2.

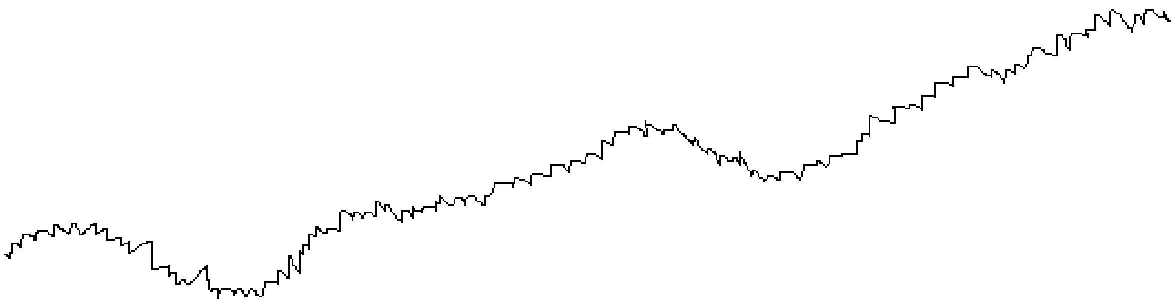

Fig.1.2. Trajectory of a system under random effects

Hence it seems reasonable to modify ODE, somehow to include the possibility of random effects disturbing the system. A formal way to do so is to write as

$$\frac{dx(t)}{dt} = f(x(t), t) + n(\mathrm{t}) \qquad t > 0 \tag{1.2}$$

$$x(0) = x_0$$

where $n(\mathrm{t})$ is the white noise. White noise is the time derivative of the Wiener process. A Wiener process $W(t)$ is a real-valued Gaussian process such that $W(0) = 0$ and the mean is zero.

$$n(\mathrm{t}) = \frac{dW(t)}{dt} \tag{1.3}$$

where $W(t)$ is the Wiener process. From eqs. (1.2) and (1.3), we get

$$\frac{dx(t)}{dt} = f(x(t), t) + \frac{dW(t)}{dt} \tag{1.4}$$

$$dx(t) = f(x(t), t)dt + dW(t) \tag{1.5}$$

This eq. (1.5) is the stochastic differential equation. Stochastic differential equation can be used to analyze different circuits. Circuit noise analysis is traditionally done in the frequency domain. In this thesis, a time-domain approach based on solving a stochastic differential equation is used. The method of SDEs in circuit noise analysis was used in [11] from a circuit simulation point of view. Their approach is based on the linearization of SDEs about its simulated deterministic trajectory.

1.3 LITERATURE REVIEW

The noise which affects the electrical system is of two types: external noise and internal noise. A well-known example of internal noise in an electrical circuit is thermal noise caused by the discreteness of electric charges [1-2]. Many other types of internal noises in electrical circuits are: shot noise, low-frequency noise, burst noise etc. When deterministic differential equations are used to do the modeling of electrical circuits, the effects of noise is ignored. Random effects due to both external and internal noise can be included by replacing the input and internal parameters in the deterministic model by random processes. Such types of random differential equations may be interpreted as stochastic differential equations [3-4]. Solutions of these equations represent Markov diffusion processes, the prototype of which is the Brownian motion process also called Wiener process [5–8]. The method of SDEs is used in paper [9] to derive and compute nonlinear, non-stationary and non-Gaussian stochastic characterizations for both amplitude and phase noise in an oscillator. A qualitative theory for the Josephson circuit family is developed in paper [10] using dissipative Hamiltonian systems theory of ordinary and stochastic differential equations. The method of SDEs in circuit noise analysis was used in paper [11] from a circuit simulation point of view. Their approach is based on the linearization of SDEs about its simulated deterministic trajectory.

Noise analysis of sampling mixer is performed in paper [12]. Three different sources of noise are analyzed. Conventional frequency domain method is used to analyze the external RF noise and intrinsic noise. Time domain method using stochastic differential equation is used to analyze the external local oscillator (LO) noise. Noise analysis of single-ended input differential amplifier is performed using stochastic differential equation in paper [13]. Various statistics of output like mean and variance is obtained using stochastic differential equation. In paper [14] application of Ito's stochastic calculus is shown to solve the problem of modeling RC circuit. Modeling of RC circuit is done to analyze the effect of external and internal noise. DC analysis of an RC circuit is performed using first order ordinary differential equation and its stochastic analogues. In the deterministic model, noise source is added in potential source and in resistance to obtain the stochastic model. Noise which is added in the potential source is assumed as a white noise. In paper [15] noise analysis of simple single stage low-pass filter (SSLPF) with the fractional-order capacitor is performed with the help of stochastic differential equation. Various solution statistics of output like mean, variance is obtained using stochastic and fractional calculus. The change in statistics with the capacitor order is investigated. It is shown that advantage of fast response can be achieved by having order of the capacitor greater than one but output noise power is higher so capacitor order should be less than unity if better noise performance is required The closed form solutions of the step response of fractional filter are obtained.

Simulation and modeling of phase noise in open loop oscillator is performed in paper [16]. A numerical methodology is presented for transistor level phase noise characterization of open loop oscillator. New technique of noise simulation which is able to compute noise is presented in paper [17]. In this technique noise sources are included in the time domain. The spectrum characteristics of these sources are related to the new models. In the new models every bias condition are considered. With this method noise simulation is possible in transient analysis. This can be applied to any kind of circuit. Jitter in ring oscillator is described and predictions are verified experimentally in paper [18]. The methodology to guide design of voltage controlled low-jitter ring oscillator is developed in this paper. Time domain figure of merit is the important design parameter, which provides the link between circuit-level design and system-level jitter. This also suggests that jitter performance of a ring depends on individual gate and not on the number of gates in the ring. In paper [19] the analysis of noise in MOS devices is presented. In paper [20] stochastic modeling of linear circuits is done by including variance in parameters. Application of the stochastic calculus in the modeling of a series RLC electric circuit is shown in this paper. To obtain the stochastic model from deterministic model, noise is added in both potential source and parameters. The noise which is added in the potential source is assumed as a white noise. Non-Monte Carlo method is used to solve the resulting SDE. A review is presented in paper [21] about the sources and characteristics of frequency fluctuations in stable oscillators. Phase noise in stable oscillator usually arises from additive voltage fluctuations and direct parameter modulation process [21].

A model is presented in paper [22] which is capable of predicting thermal noise in MOSFET. The model presented in this paper predicts the behaviors of thermal noise of both short and long channel devices accurately. [23] provides the introduction to the problem of noise from the circuit design point of view. A Monte Carlo method for circuit simulation is presented in paper [24]. Noise analysis of phase locked loop is performed in paper [25]. Stochastic differential equation is used to formulate the problem and techniques are discussed to obtain the solution. In paper [26] stochastic circuit modeling is done with hermite polynomial chaos. The proposed methodology gives better accuracy of statistical description than root-sum-square method for random temperature influence analysis of a band pass filter. In paper [27] efficient method for electronic circuit noise performance calculation is used. This method allow to consider large number of uncorrelated noise sources. [28, 29] are referred for the equivalent circuit of the active devices. A single-ended differential input amplifier is used as the initial amplification stage of a preamplifier used in the read channel of a HDD application [30]. Extrinsic noise enters into the read channels of the preamplifier from the substrate capacitances of the input transistors connected to the red heads [30]. [31-42] provided the knowledge about stochastic process, stochastic differential equation, differential equation and random noise. A model which is capable to predict noise accurately in electrical oscillators is introduced in paper [43]. Approach presented in this paper shows that the total phase noise is contributed by the noise located near integer multiples of the oscillation frequency. In paper [44] the analysis of white noise in oscillator is done. Noise is of major concern in oscillators, because small noise into oscillator leads to change in its frequency spectrum and timing property [45].A solid foundation for phase noise is developed in paper [45] that is valid for any oscillator, regardless of operating mechanism. Nobel results are established about the dynamic of stable nonlinear oscillators in the presence of perturbations. Nonlinear equation is obtained for phase error, which is solved without approximations for random perturbations.

Oscillator noise analysis is performed in paper [46]. Phase noise analysis in oscillator is performed in paper [47] with colored noise source. Computer simulation of low frequency 1/f noise performance of electronic circuit is described in paper [48]. A numerical noise analysis method is proposed in paper [49] for nonlinear circuits with periodic large signal excitation. A non-linear circuit is modeled as a linear periodic time-varying circuit, for the small signal input response. For small signal characteristics and noise figure for the linear periodic time-varying circuit, calculation methods have been explained in paper [49]. Theory and practical characterization of phase noise in oscillator due to colored, as opposed to white, noise sources is presented in paper [50]. A theory for the noise analysis performance in mixer is presented in paper [51]. The implementation of this theory is also included in paper [51]. This theory is utilized to analyze the noise figure. An evaluation of the performance of a coherent receiver has carried out in paper [52]. It is shown that the AM noise due to the laser working as local oscillator degrades the expected SNR. Numerical approach is presented in paper [53] to statistically characterize filtered phase noise which encounter in heterodyne optical fiber receiver. The importance of results in paper [53] is twofold. First, they can be used in error rate analysis of heterodyne optical receivers. Second, they provide a general statistical representation which is useful for large and small phase noise.

In paper [54] accurate and detailed noise calculation using MATLAB is proposed for operational amplifier circuits. Total equivalent input spot noise voltage, total output spot noise voltage, noise figure, integrated noise with its two components: 1/f noise voltage and equivalent white noise voltage as well as noise power are calculated in paper [54]. Noise performance study of low noise amplifier is presented in paper [55] in small and large signal conditions. Measurements show that the LNA NF can increase dramatically with the gain compression level. AM and PM noise analysis in quartz crystal oscillators is performed in paper [56]. An algorithm to obtain ordinary differential equation of oscillator behavior, including individual noise sources is proposed in paper [56]. A method is proposed in paper [57] for noise analysis of dual gate FET mixers. Simplified model for DGFET as a cascade of two FETs is used in this method. Then noise correlation matrix of the DGFET mixer is calculated by applying proper noise sources, by two noise temperature model. Noise analysis of bipolar harmonics mixer for use with direct conversion receivers is presented in paper [58]. Shot noise and thermal noise are modeled as amplitude modulated white noises, which are considered as cyclostationary noises. To analyze the time-domain noise performance of linear time- invariant and linear time-variant circuits, a custom simulation tool that combines MATLAB and HSPICE is presented in paper [59]. In this digital filter architecture is presented which generates physically realistic 1/f-noise signals over short time intervals.

In [60] an approach of noise analysis of FET oscillator is presented. Internal noise sources are introduced as random signals in the time-domain. Using the simulation approach explained the elements that most strongly effect phase noise can be separately identified and studied, and can help to develop circuit designs with better optimize phase noise performance in FET-based oscillators. A numerical technique for time domain noise analysis of oscillator is presented in paper [61]. An approach is presented in paper [62] to characterize low frequency noise for semiconductor devices. Using this technique, performance of different devices can be compared. Modeling of RL circuit is done in [63] using SDE. Stochastic model is obtained for the circuit by adding a noise term in both the source and the resistance. The analytic solution for the obtained SDE is presented. A technique is presented in paper [64] for the analysis of higher order electrical circuits excited from the random sources using SDE. The current and voltage responses of the circuits are obtained. To analyze the response of the circuit models of optional order, consisting of a cascade connection of the RLGC networks, this method is applied. In paper [65] application of the SDE to the problem of modeling RLC circuits is presented. The stochastic model is obtained from deterministic model by adding a noise term to various parameters of the circuit. The analytic solutions of obtained SDEs are found.

The nonlinear stochastic differential equation-generating power-law distributed signal and 1/f noise are considered in paper [66]. The analysis shows that the power spectrum may be represented as a sum of the Lorentzian spectra and provides further insights into the origin of 1/f noise. Stochastic model of 1/f noise based on the linear stochastic differential equation with the very slowly varying coefficients or consisting of a superposition of uncorrelated components with different distribution of these coefficients is considered in paper [67]. Stochastic behaviours of Tow-Thomas biquadratic filter is analysed using stochastic differential equation in paper [68]. Noise in the applied voltage source is considered as white noise. Stochastic behaviour of the filter is analysed from the obtained solution. A noise analysis method based on Multisim is presented in paper [69]. Multisim creates a noise model circuit, using noise models of each resistor and semiconductor device, instead of AC models, then performs analysis. It calculates the noise contribution of each resistor and other semiconductor devices at the output node. The static noise margin model for pseudo-CMOS logic circuit is derived in paper [70]. The impact of design parameters on noise margin is analysed. [71] deals with the fabrication of an insulatorless wideband low-noise amplifier. Low- noise amplifier includes two branches in parallel: a common-source path and a common-gate path. To eliminate the noise contribution, the noise cancellation technique is applied. The noise figure is improved. Noise analysis of switched capacitor unity gain sampler is presented in paper [72]. The noise analysis method starts from establishing noise model for the sampler in each clock pulse. Then output noise power contributed by each noise source is derived, based on the characteristic of charge transferring by capacitor and theory of random process. Noise analysis of CMOS inverter is done in paper [73] using SDE. In this CMOS inverter stochastic differential equation is developed.

On the basis of literature review, it has observed that the work can be proposed on the noise analysis of different BJT amplifiers, FET amplifiers and MOS differential amplifier.

1.4 RESEARCH OBJECTIVES

The objectives of the proposed research work are:

1. To do the noise analysis of different BJT amplifier configurations.
2. To do the noise analysis of different FET amplifiers configurations.
3. To do the noise analysis of differential amplifier (MOS).
4. To do the noise analysis of different amplifiers with variable load and capacitive load.

1.5 CONTRIBUTION IN THE THESIS

Different BJT and FET amplifiers have various important applications, so their noise analysis is very important. In superheterodyne receiver intermediate frequency (IF) amplifier is used after mixer. IF amplifier consists of bipolar junction transistor or field effect transistor amplifier. Radio frequency (RF) amplifier is also used in receiver which consist of bipolar transistor or field effect transistor. FET or BJT stage is used in the mixer of the receiver. In amplitude modulation receivers and frequency modulation receivers, RF amplifier, mixer and IF amplifier are used.

Differential amplifier is very important circuit in analog circuit design. Operational amplifier consists of differential amplifier. The differential amplifier is less affected by noise if it works on dual input mode as compare to single ended input mode. Differential amplifier in single ended input mode is used in hard disk drive (HDD) application [30]. Preamplifier is used in hard disk drive application. Initial amplification in preamplifier is done using single ended input differential amplifier. Thus noise analysis of differential amplifier in single ended input mode is very important.

In chapter 2, the noise analysis of different BJT amplifiers like common base amplifier, common collector amplifier and common emitter amplifier is performed. The mean and variance is determined for the circuits and observed that the noise affects the considered circuits more at high input frequencies. The results are compared with the Monte Carlo simulation results. These circuits are also analysed for variable load resistance. In this thesis, in chapter 3 different FET amplifiers like common source amplifier, common drain amplifier and common gate amplifier are considered to do their noise analysis. First and second order statistics are obtained for the output voltage for the circuits and found out that the noise performance of the circuit degraded at high input frequencies. The results are compared with the Monte Carlo simulation results. Noise analysis is also performed for variable load resistance. Common source and common gate amplifier are analysed for capacitive load too.

Investigation into the noise analysis of MOS differential amplifier is done in chapter 4. This analysis leads us to the conclusion that the circuit becomes more sensitive to noise at high input frequencies. Circuit is also considered for variable load resistance. Then the effects of device parameters are investigated on the noise performance of MOS differential amplifier. Then the noise performance of the BJT and MOS differential amplifier is compared.

CHAPTER 2

NOISE ANALYSIS OF BJT AMPLIFIER CONFIGURATIONS USING SDE

2.1 INTRODUCTION

Noise analysis of different BJT amplifiers is done in this chapter. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise on different BJT amplifiers is analysed. In this thesis, time domain method based on solving stochastic differential equation is used. To derive and compute non-Gaussian, non-stationary and nonlinear stochastic characterization of both amplitude and phase noise in an oscillator, the stochastic differential equation approach is adopted in [9]. The stochastic differential equation approach was adopted in [11] from simulation point of view for noise analysis. This method is based on linearization of stochastic differential equation about its simulated deterministic trajectory. In [12] noise analysis of sampling mixer is done. Three different sources of noise are analyzed. Conventional frequency domain method is used to analyze the external RF noise and intrinsic noise. Time domain method using stochastic differential equation is used to analyze the external local oscillator (LO) noise. In [13] noise analysis of single-ended input differential amplifier is performed using stochastic differential equation. Various statistics of output like mean and variance is obtained using stochastic differential equation. In [14] modeling of RC circuit is done to analyze the effect of external and internal noise. DC analysis of an RC circuit is performed using first order ordinary differential equation and its stochastic analogues. In [15] noise analysis of simple single stage low-pass filter (SSLPF) with the fractional-order capacitor is performed with the help of stochastic differential equation. Various solution statistics of output like mean, variance is obtained using stochastic and fractional calculus. The change in statistics with the capacitor order is investigated. The closed form solutions of the step response of fractional filter are obtained.

The noise is assumed to be white Gaussian noise. Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. Since external noise analysis has to be done so time domain method is used which involves solving the stochastic differential equation.

2.2 NOISE ANALYSIS OF COMMON-BASE AMPLIFIER USING STOCHASTIC DIFFERENTIAL EQUATION

The common-base amplifier is an important circuit in analog design. There are varieties of applications of common-base amplifier. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise on the common-base amplifier is analysed. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although it is an ideal condition, when noise is assumed to be white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Noise analysis is generally performed in frequency domain. If the circuit is linear and time invariant, this method is useful. The system can be either non-linear or time-variance if the noise analysis is done for the external noise. Therefore frequency domain method is not useful for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common-base amplifier. The autocorrelation function of the output noise and other statistics like mean and variance is obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

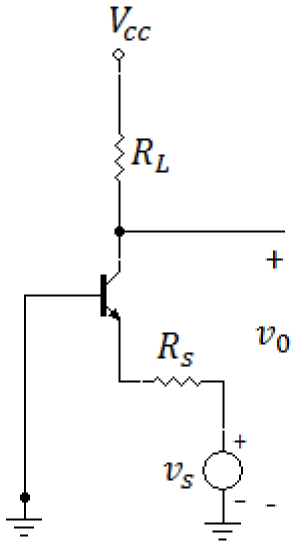

Fig. 2.1. Common-Base amplifier

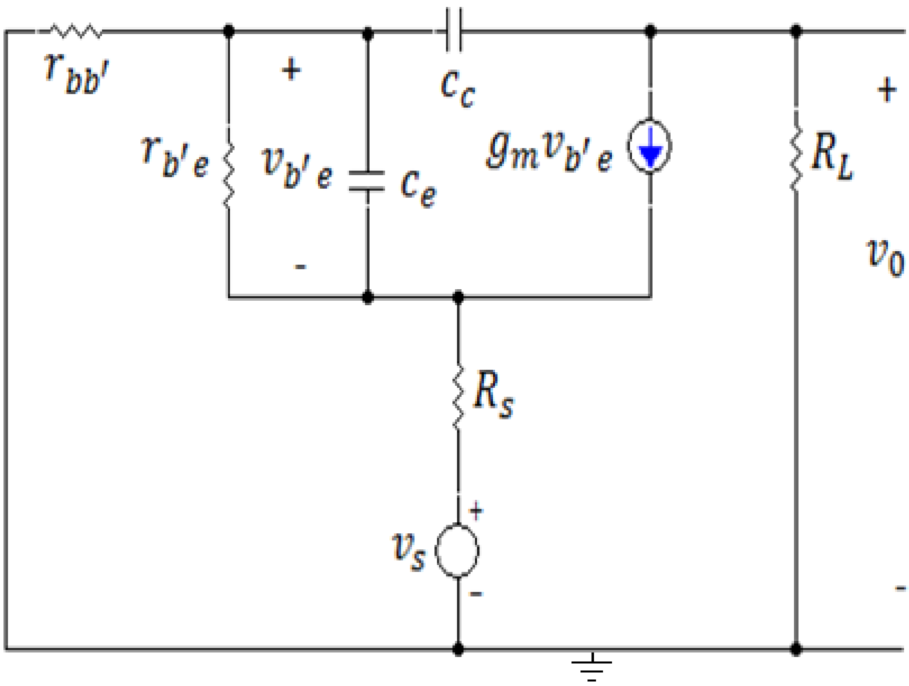

Fig. 2.2. High-Frequency Equivalent Circuit of CB amplifier

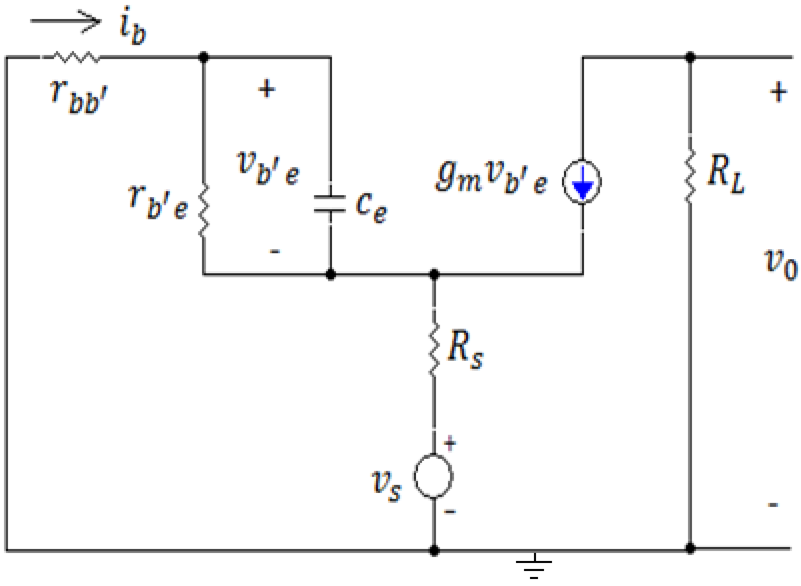

Fig. 2.3. Simplified High-Frequency Equivalent Circuit of CB amplifier

2.2.1 Analysis of Noise using SDE

Common-base amplifier is considered which is shown in Fig. 2.1. Fig. 2.2 represents its high-frequency equivalent circuit. We can obtain simplified equivalent circuit using Miller's Theorem as shown in Fig. 2.3 [28]. Now, we will analyse this circuit using SDEs. From the simplified equivalent circuit, we obtain

$$i_b(t) = \frac{v_{b'e}(t)}{r_{b'e}} + c_e \frac{dv_{b'e}(t)}{dt} \tag{2.1}$$

and

$$-i_b(t) r_{bb'} - v_{b'e}(t) - R_s\big(i_b(t) + g_m v_{b'e}(t)\big) - v_s(t) = 0$$

$$i_b(t) = -\frac{v_s(t)}{R_{s'}} - \frac{v_{b'e}(t)(1+g_m R_s)}{R_{s'}} \tag{2.2}$$

where $R_s' = R_s + r_{bb'}$. From eqs. (2.1) and (2.2), we get

$$\frac{dv_{b'e}(t)}{dt} + k v_{b'e}(t) = -\frac{v_s(t)}{c_e R_{s'}} \tag{2.3}$$

where $k = \frac{1}{c_e}\left(\frac{1+g_m R_s}{R_{s'}} + \frac{1}{r_{b'e}}\right)$ and

$$v_0(t) = -g_m R_L v_{b'e}(t) \tag{2.4}$$

Consider $v_s(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_s(t) = \sigma n(t)$ in eq. (2.3)

$$\frac{dv_{b'e}(t)}{dt} + kv_{b'e}(t) = -\frac{\sigma n(t)}{c_e R_{S'}}$$

$$dv_{b'e}(t) + kv_{b'e}(t)dt = -\frac{\sigma n(t)dt}{c_e R_{S'}}$$

Then we put $n(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_{b'e}(t) + kv_{b'e}(t)dt = -\frac{\sigma dW(t)}{c_e R_{S'}} \tag{2.5}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_{b'e}(t)$. We take the expectation of both side of eq. (2.5)

$$dE[v_{b'e}(t)] + kE[v_{b'e}(t)]dt = -\frac{E[\sigma dW(t)]}{c_e R_{S'}} \tag{2.6}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (2.6) we obtain

$$\frac{dE[v_{b'e}(t)]}{dt} + kE[v_{b'e}(t)] = 0 \tag{2.7}$$

The solution of eq. (2.7) is written as

$$E[v_{b'e}(t)] = c_1 e^{-kt} \tag{2.8}$$

This is the mean of $v_{b'e}(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. From eqs. (2.4) and (2.8) we can obtain the mean of the output

$$E[v_0(t)] = -g_m R_L\, E[v_{b'e}(t)]$$

$$E[v_0(t)] = -g_m R_L c_1 e^{-kt} \tag{2.9}$$

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eq. (2.3) is rewritten

$$\frac{dv_{b'e}(t)}{dt} + kv_{b'e}(t) = -\frac{v_s(t)}{c_e R_s'} \tag{2.10}$$

Eq. (2.10) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_{b'e}(t)$ are zero at $t_1 = 0$. Both sides of eq. (2.10) is multiplied by $v_{b'e}(t_2)$ and then expectation is taken

$$\frac{dR_{v_{b'e},v_{b'e}}(t_1,t_2)}{dt_1} + kR_{v_{b'e},v_{b'e}}(t_1,t_2) = -\frac{R_{v_s,v_{b'e}}(t_1,t_2)}{c_e R_s'} \tag{2.11}$$

Again, eq. (2.10) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_s(t)$ and $v_{b'e}(t)$ are zero at $t_2 = 0$. Both sides of eq. (2.10) is multiplied by $v_s(t_1)$ and then expectation is taken

$$\frac{dR_{v_s,v_{b'e}}(t_1,t_2)}{dt_2} + kR_{v_s,v_{b'e}}(t_1,t_2) = -\frac{R_{v_s,v_s}(t_1,t_2)}{c_e R_s'} \tag{2.12}$$

We know that $R_{v_s,v_s}(t_1,t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (2.12) is given as

$$R_{v_s,v_{b'e}}(t_1,t_2) = -\frac{\sigma^2}{c_e R_s'} e^{k(t_1-t_2)} \tag{2.13}$$

Now, we put the value of $R_{v_s,v_{b'e}}(t_1,t_2)$ from eq. (2.13) in eq. (2.11), we obtain

$$R_{v_{b'e},v_{b'e}}(t_1,t_2) = \frac{\sigma^2}{2k(c_e R_s')^2}\left(e^{-k(t_1-t_2)} - e^{-k(t_1+t_2)}\right) \tag{2.14}$$

When we substitute $t_1 = t_2 = t$ in eq. (2.14), we get the second order moment of $v_{b'e}(t)$.

$$E[{v_{b'e}}^2(t)] = \frac{\sigma^2}{2k(c_e R_s')^2}(1 - e^{-2kt}) \tag{2.15}$$

From eqs. (2.4) and (2.15) we will have the second order moment of $v_o(t)$ that is variance of the output in this case.

$$E[{v_0}^2(t)] = (g_m R_L)^2 E[{v_{b'e}}^2(t)]$$

$$E[{v_0}^2(t)] = \frac{(g_m R_L)^2\sigma^2}{2k(c_e R_s')^2}(1 - e^{-2kt}) \tag{2.16}$$

2.2.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_L = 10^4\Omega$, $R_s = 5 \times 10^3\Omega$, $r_{bb'} = 100\Omega$, $r_{b'e} = 1.5 \times 10^3\Omega$, $c_e = 2pF$, $\sigma = 0.25$, $g_m = 40mA/V$.

Fig. 2.4 represents the variation of mean of output voltage with time for non-zero initial conditions ($v_{b'e}(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 2.4 that the magnitude of mean of the output has peak value 4 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 1μs (7 μs in case of single ended input BJT differential amplifier [13]). Fig. 2.5 represents the variation of variance of output with time. After increasing linearly with time, variance becomes constant. We also analysed the circuit for variable load resistance. Fig. 2.6 represents the variation of variance with load resistance. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 1MHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 10 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz. The standard tool for the simulation with random input is Monte Carlo simulation. We compare our results with Monte Carlo simulation. Fig. 2.7 shows the comparison of deterministic, Monte Carlo and stochastic solution. Fig 2.7 shows that result of Monte Carlo simulation is very close to the stochastic solution and deterministic solution (The stochastic solution is very close to deterministic solution in [14] and deterministic solution is very close to stochastic and Monte Carlo solution in [20]).

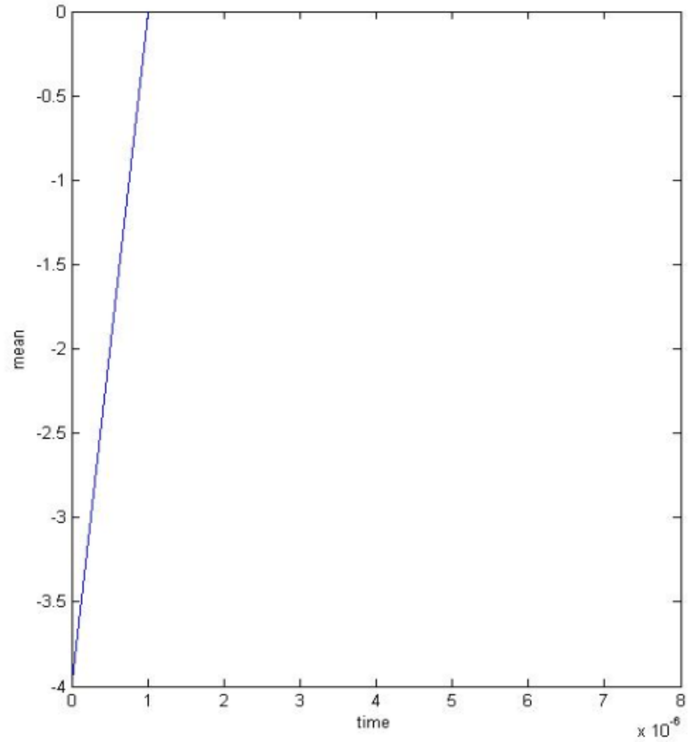

Fig.2.4. Variation of mean of the output with time for CB amplifier

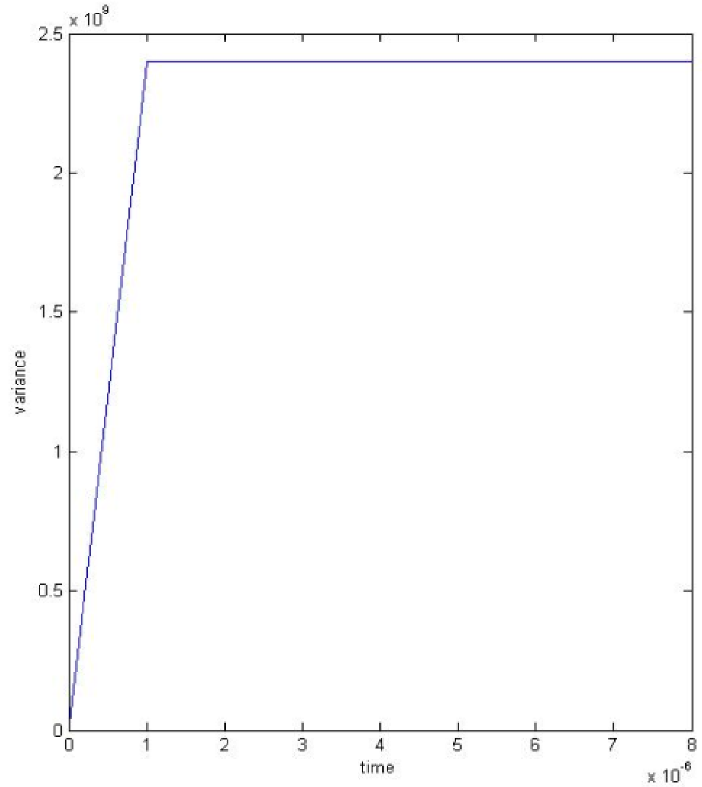

Fig.2.5. Variation of variance of the output with time for CB amplifier

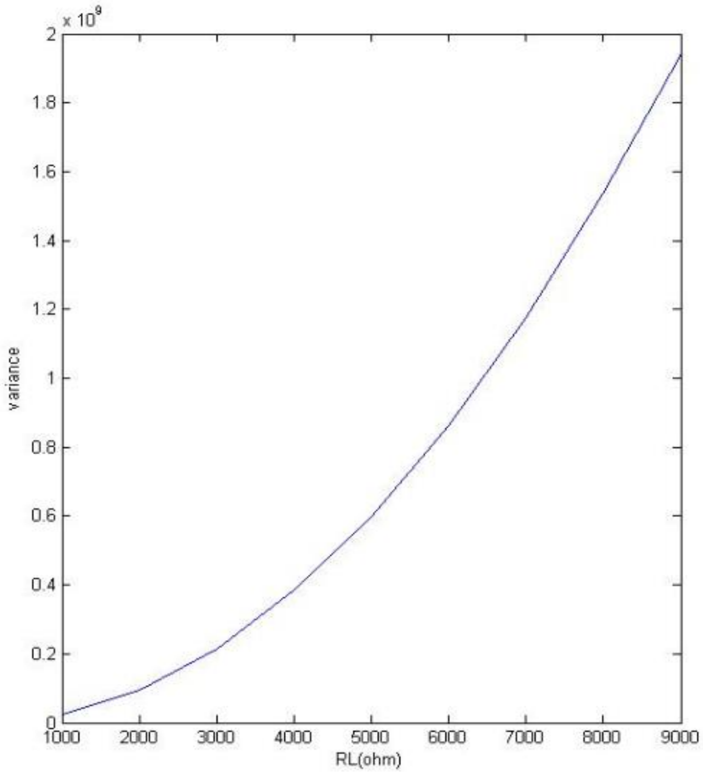

Fig.2.6. Variation of variance of the output with R_L for CB amplifier

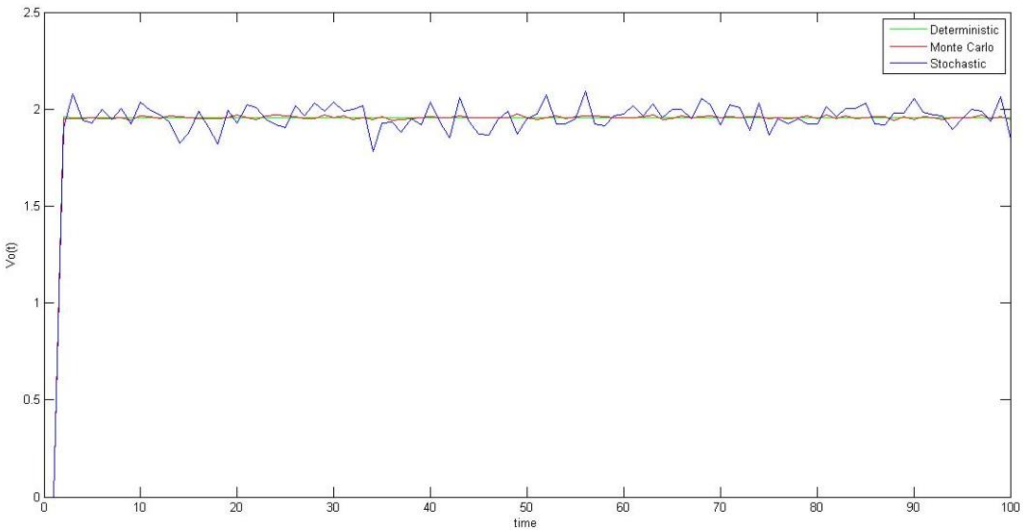

Fig.2.7. Comparison of deterministic, stochastic and Monte Carlo simulation for CB amplifier

2.3 NOISE ANALYSIS OF COMMON-COLLECTOR AMPLIFIER USING STOCHASTIC DIFFERENTIAL EQUATION

The common-collector amplifier is an important circuit in analog design. There are varieties of applications of common-collector amplifier. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common-collector amplifier. We analysed the effect of noise at high frequencies.

The noise is assumed to be white Gaussian noise. Although It is an ideal condition, when noise is white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common-collector amplifier. The autocorrelation function of the output noise and other statistics like mean and variance are obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

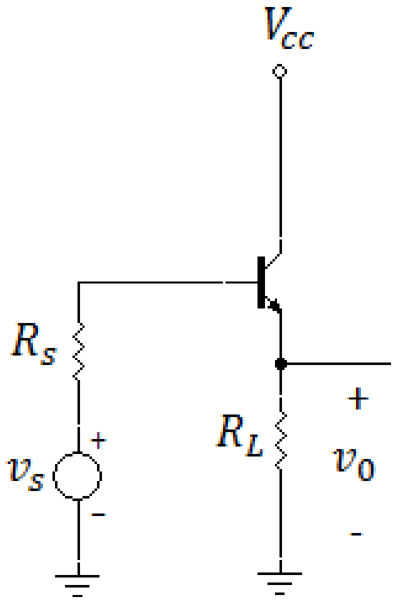

Fig. 2.8. Common-Collector Amplifier

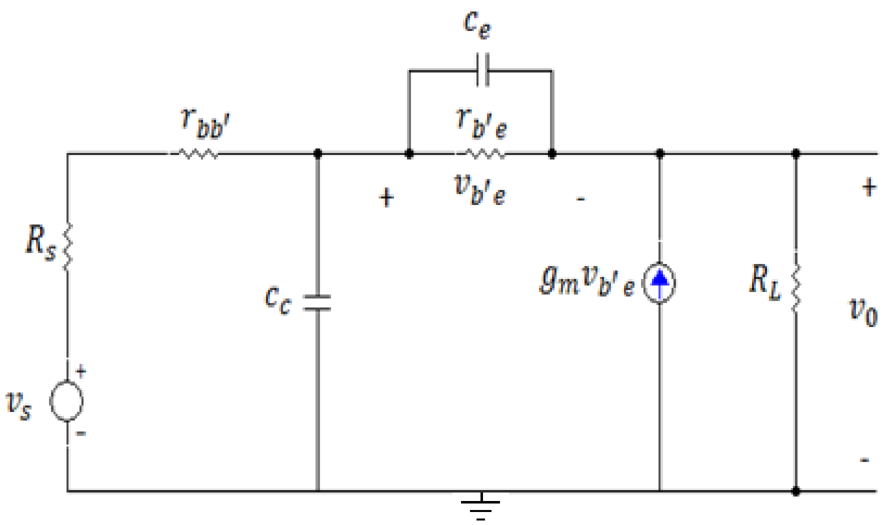

Fig. 2.9. High-Frequency Equivalent Circuit of CC amplifier

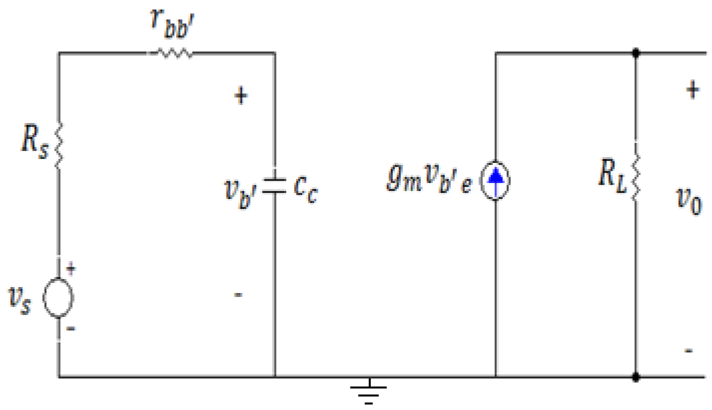

Fig. 2.10. Simplify High-Frequency Equivalent Circuit of CC amplifier

2.3.1 Analysis of Noise using SDE

Common-collector amplifier is considered which is shown Fig. 2.8. Fig. 2.9 represents its high-frequency equivalent circuit. By Miller's theorem, c_e and $g_{b'e}$ can be transferred into input side by $c_e(1-k)$ and $g_{b'e}(1-k)$ and into output side by $c_e\,(k-1)/k$ and $g_{b'e}\,(k-1)/k$. Where $g_{b'e} = 1/r_{b'e}$. As the gain (k) for the common-collector amplifier is approximately 1, the value of $(1-k)$ is close to zero. So this approximation lead us to obtain the simplified high-frequency equivalent circuit as shown in Fig. 2.10.Now, we will analyse this circuit using SDE. From the simplified equivalent circuit, we obtain

$$\frac{v_s(t)-v_{b'}(t)}{R_{s'}} = c_c\frac{dv_{b'}(t)}{dt} \tag{2.17}$$

where $R_s' = R_s + r_{bb'}$. Eq. (2.17) can be simplified further and written as

$$\frac{dv_{b'}(t)}{dt} + k_1 v_{b'}(t) = \frac{v_s(t)}{c_c R_{s'}} \tag{2.18}$$

where $k_1 = \frac{1}{c_c R_{s'}}$

$$v_0(t) = v_e(t) = g_m R_L v_{b'e}(t)$$

$$v_0(t) = v_e(t) = \frac{g_m R_L}{1+g_m R_L} v_{b'}(t) \tag{2.19}$$

Consider $v_s(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_s(t) = \sigma n(t)$ in eq. (2.18)

$$\frac{dv_{b'}(t)}{dt} + k_1 v_{b'}(t) = \frac{\sigma n(t)}{c_c R_{s'}}$$

$$dv_{b'}(t) + k_1 v_{b'}(t)dt = \frac{\sigma n(t)dt}{c_c R_{s'}}$$

Then we put $n(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_{b'}(t) + k_1 v_{b'}(t)dt = \frac{\sigma dW(t)}{c_c R_{s'}} \tag{2.20}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_{b'}(t)$. We take the expectation of both side of eq. (2.20)

$$dE[v_{b'}(t)] + k_1 E[v_{b'}(t)]dt = \frac{E[\sigma dW(t)]}{c_c R_{s'}} \tag{2.21}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (2.21) we obtain

$$\frac{dE[v_{b'}(t)]}{dt} + k_1 E[v_{b'}(t)] = 0 \tag{2.22}$$

The solution of eq. (2.22) is written as

$$E[v_{b'}(t)] = c_1 e^{-k_1 t} \tag{2.23}$$

This is the mean of $v_{b'}(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. From eqs. (2.19) and (2.23) we can obtain the mean of the output

$$E[v_0(t)] = E[v_e(t)] = \frac{g_m R_L}{1+g_m R_L} E[v_{b'}(t)]$$

$$E[v_0(t)] = \frac{g_m R_L}{1+g_m R_L} c_1 e^{-k_1 t} \tag{2.24}$$

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eq. (2.18) is rewritten

$$\frac{dv_{b'}(t)}{dt} + k_1 v_{b'}(t) = \frac{v_s(t)}{c_c R_{s'}} \tag{2.25}$$

Eq. (2.25) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_{b'}(t)$ are zero at $t_1 = 0$. Both sides of eq. (2.25) is multiplied by $v_{b'}(t_2)$ and then expectation is taken

$$\frac{dR_{v_{b'},v_{b'}}(t_1,t_2)}{dt_1} + k_1 R_{v_{b'},v_{b'}}(t_1,t_2) = \frac{R_{v_s,v_{b'}}(t_1,t_2)}{c_c R_{s'}} \tag{2.26}$$

Again, eq. (2.25) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_s(t)$ and $v_{b'}(t)$ are zero at $t_2 = 0$. Both sides of eq. (2.25) is multiplied by $v_s(t_1)$ and then expectation is taken

$$\frac{dR_{v_s,v_{b'}}(t_1,t_2)}{dt_2} + k_1 R_{v_s,v_{b'}}(t_1,t_2) = \frac{R_{v_s,v_s}(t_1,t_2)}{c_c R_{s'}} \tag{2.27}$$

We know that $R_{v_s,v_s}(t_1,t_2) = \sigma^2 \delta(t_1 - t_2)$, the solution of eq. (2.27) is given as

$$R_{v_s,v_{b'}}(t_1,t_2) = \frac{\sigma^2}{c_c R_{s'}} e^{k_1(t_1-t_2)} \tag{2.28}$$

Now, we put the value of $R_{v_s,v_{b'}}(t_1,t_2)$ from eq. (2.28) in eq. (2.26), we obtain

$$R_{v_{b'},v_{b'}}(t_1,t_2) = \frac{\sigma^2}{2k_1\left(c_c R_s'\right)^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \tag{2.29}$$

When we substitute $t_1 = t_2 = t$ in eq. (2.29), we get the second order moment of $v_{b'}(t)$.

$$E[{v_{b'}}^2(t)] = \frac{\sigma^2}{2k_1\left(c_c R_s'\right)^2}\left(1 - e^{-2k_1 t}\right) \tag{2.30}$$

From eqs. (2.19) and (2.30) we will have the second order moment of $v_o(t)$ that is variance of the output in this case.

$$E[{v_0}^2(t)] = E[{v_e}^2(t)] = \frac{(g_m R_L)^2}{(1+g_m R_L)^2} E[{v_{b'}}^2(t)]$$

$$E[{v_0}^2(t)] = \frac{(g_m R_L)^2 \sigma^2}{(1+g_m R_L)^2 2k_1\left(c_c R_s'\right)^2}\left(1 - e^{-2k_1 t}\right) \tag{2.31}$$

2.3.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_L = 10^4\Omega$, $R_s = 5 \times 10^3\Omega$, $r_{bb'} = 100\Omega$, $c_c = 0.8pF$, $\sigma = 0.25$, $g_m = 40mA/V$.

Fig. 2.11 represents the variation of mean with time for non-zero initial conditions ($v_{b'}(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 2.11 that the magnitude of mean of the output has peak value 0.01 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 1μs (7 μs in case of single ended input BJT differential amplifier [13]). Fig. 2.12 represents the variation of variance with time. After increasing linearly with time, variance becomes constant. We also analysed the circuit for variable load resistance. Fig. 2.13 represents the variation of variance with load resistance. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 1MHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 10 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz. The standard tool for the simulation with random input is Monte Carlo simulation. We compare our results with Monte Carlo simulation. Fig. 2.14 shows the comparison of deterministic, Monte Carlo and stochastic solution. Fig 2.14 shows that result of Monte Carlo simulation is very close to the stochastic solution and deterministic solution (The stochastic solution is very close to deterministic solution in [14] and deterministic solution is very close to stochastic and Monte Carlo solution in [20]).

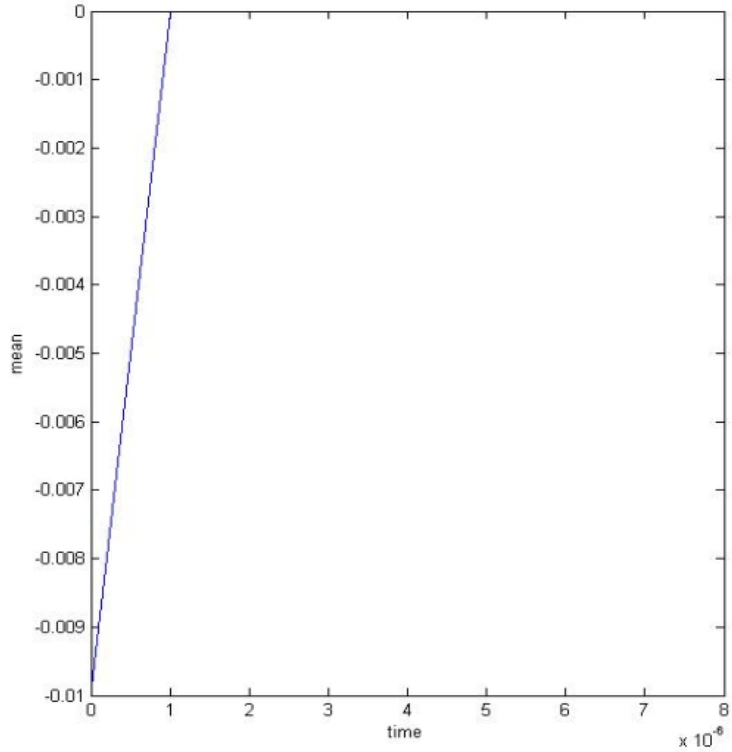

Fig.2.11. Variation of mean with time for CC amplifier

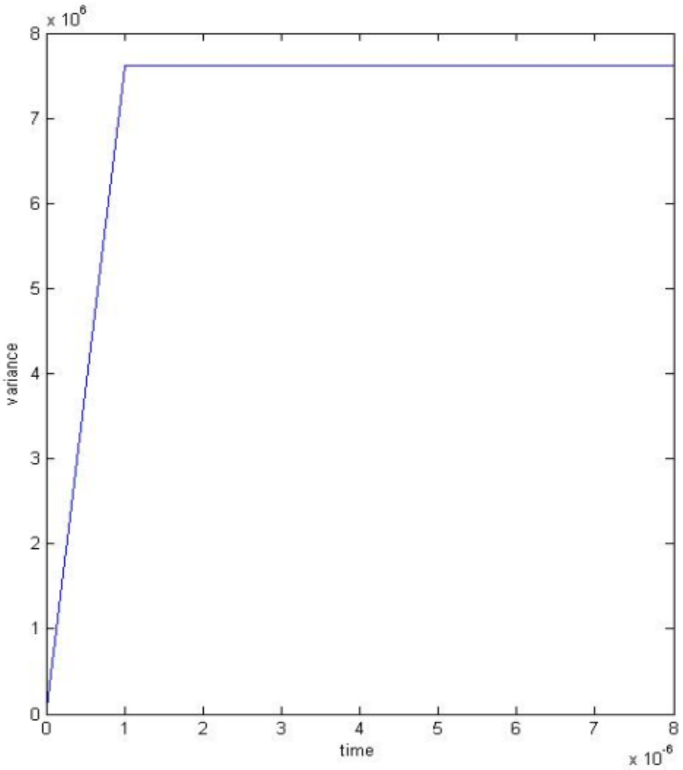

Fig.2.12. Variation of variance with time for CC amplifier

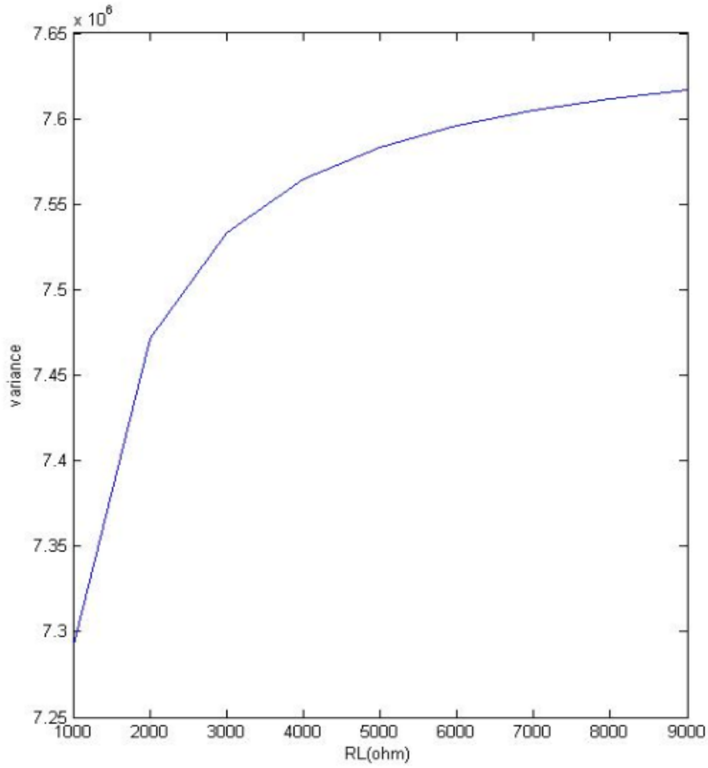

Fig.2.13. Variation of variance with R_L for CC amplifier

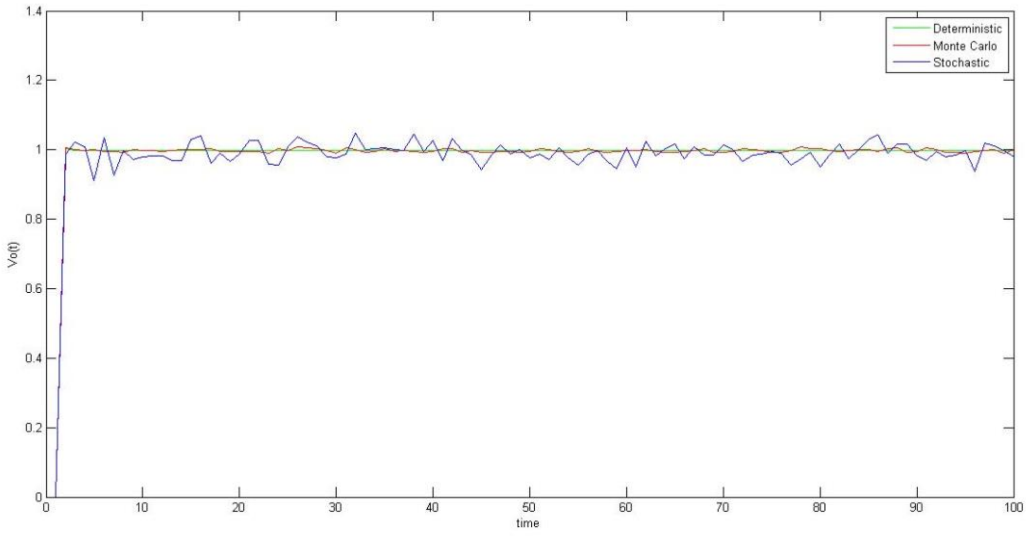

Fig.2.14. Comparison of deterministic, stochastic and Monte Carlo simulation for CC amplifier

2.4 NOISE ANALYSIS OF COMMON-EMITTER AMPLIFIER USING STOCHASTIC DIFFERENTIAL EQUATION

The common-emitter amplifier is an important circuit in analog design. There are varieties of applications of common- emitter amplifier. The noise which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common- emitter amplifier. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although It is an ideal condition, when the noise is assumed to be white Gaussian, It can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common- emitter amplifier. The autocorrelation function of the output noise and other statistics like mean and variance are obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

2.4.1 Analysis of noise using SDE

Common-emitter amplifier is considered which is shown in Fig. 2.15. Fig. 2.16 represents its high-frequency equivalent circuit. We can obtain simplified equivalent circuit using Miller’s Theorem as shown in Fig. 2.17. Now, we will analyse this circuit using SDE.

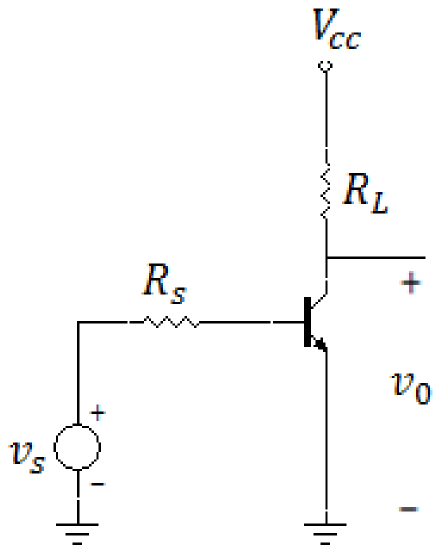

Fig. 2.15. Common-Emitter Amplifier

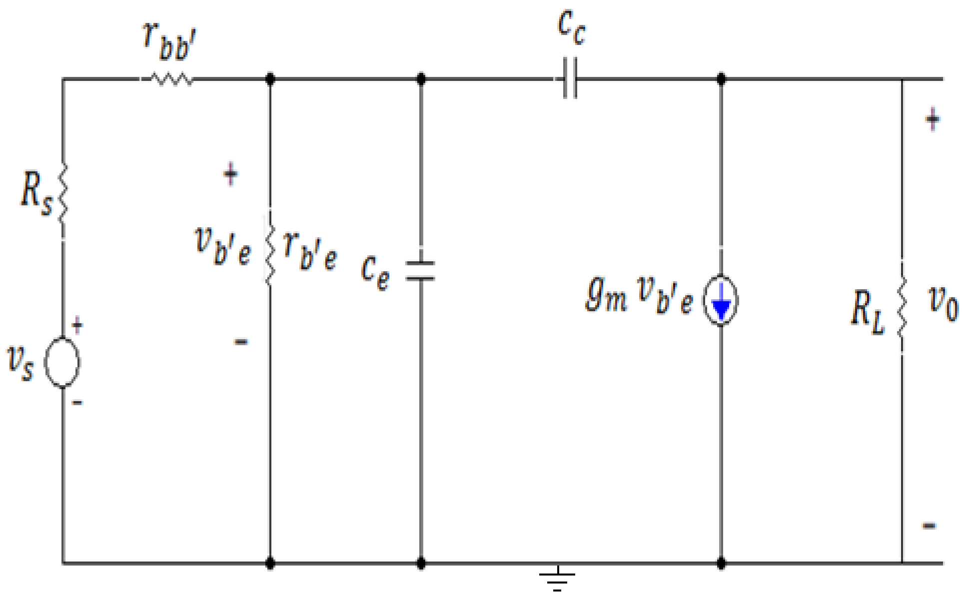

Fig. 2.16. High-Frequency Equivalent Circuit of CE amplifier

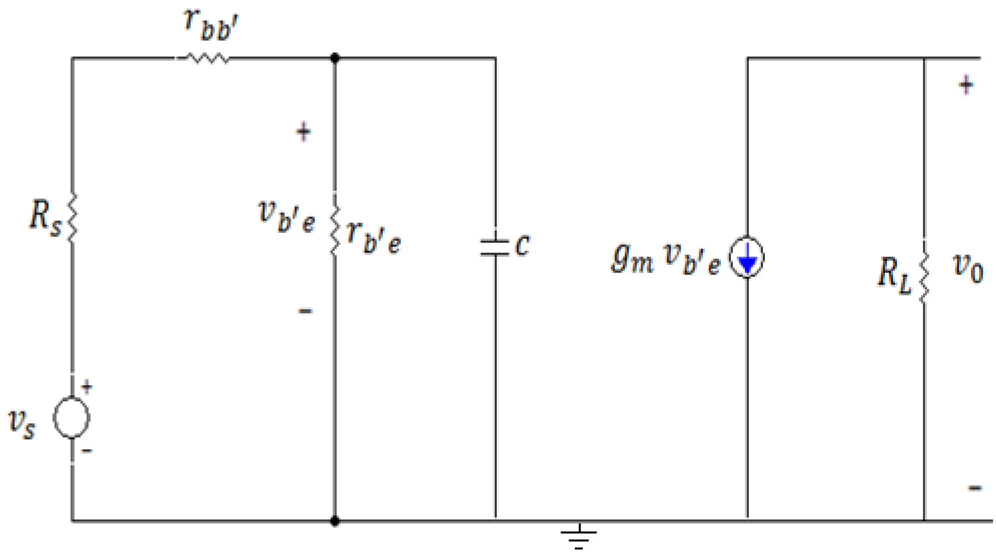

Fig. 2.17. Simplified High-Frequency Equivalent Circuit of CE amplifier

From the simplified equivalent circuit, we obtain

$$\frac{v_s(t)-v_{b'e}(t)}{R_{s'}} = \frac{v_{b'e}(t)}{r_{b'e}} + c\,\frac{dv_{b'e}(t)}{dt} \tag{2.32}$$

where $c = c_e + c_c(1 + g_m R_L)$ & $R_s' = R_s + r_{bb'}$. Eq. (2.32) can be simplified further and written as

$$\frac{dv_{b'e}(t)}{dt} + kv_{b'e}(t) = \frac{v_s(t)}{cR_{s'}} \tag{2.33}$$

where $k = \frac{1}{c}(\frac{1}{R_{s'}} + \frac{1}{r_{b'e}})$ and

$$v_0(t) = -g_m R_L v_{b'e}(t) \tag{2.34}$$

Consider $v_s(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_s(t) = \sigma n(t)$ in eq. (2.33)

$$\frac{dv_{b'e}(t)}{dt} + kv_{b'e}(t) = \frac{\sigma n(t)}{cR_{s'}}$$

$$dv_{b'e}(t) + kv_{b'e}(t)dt = \frac{\sigma n(t)dt}{cR_{s'}}$$

Then we put $n(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_{b'e}(t) + kv_{b'e}(t)dt = \frac{\sigma dW(t)}{cR_{s'}} \tag{2.35}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_{b'e}(t)$. We take the expectation of both side of eq. (2.35)

$$dE[v_{b'e}(t)] + kE[v_{b'e}(t)]dt = \frac{E[\sigma dW(t)]}{cR_{s'}} \tag{2.36}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (2.36) we obtain

$$\frac{dE[v_{b'e}(t)]}{dt} + kE[v_{b'e}(t)] = 0 \tag{2.37}$$

The solution of eq. (2.37) is written as

$$E[v_{b'e}(t)] = c_1 e^{-kt} \tag{2.38}$$

This is the mean of $v_{b'e}(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. From eqs. (2.34) and (2.38) we can obtain the mean of the output

$$E[v_0(t)] = -g_m R_L\, E[v_{b'e}(t)]$$

$$E[v_0(t)] = -g_m R_L c_1 e^{-kt} \tag{2.39}$$

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eq. (2.33) is rewritten

$$\frac{dv_{b'e}(t)}{dt} + kv_{b'e}(t) = \frac{v_s(t)}{cR_{s'}} \tag{2.40}$$

Eq. (2.40) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_{b'e}(t)$ are zero at $t_1 = 0$. Both sides of eq. (2.40) is multiplied by $v_{b'e}(t_2)$ and then expectation is taken

$$\frac{dR_{v_{b'e},v_{b'e}}(t_1,t_2)}{dt_1} + kR_{v_{b'e},v_{b'e}}(t_1,t_2) = \frac{R_{v_s,v_{b'e}}(t_1,t_2)}{cR_{s'}} \tag{2.41}$$

Again, eq. (2.40) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_s(t)$ and $v_{b'e}(t)$ are zero at $t_2 = 0$. Both sides of eq. (2.40) is multiplied by $v_s(t_1)$ and then expectation is taken

$$\frac{dR_{v_s,v_{b'e}}(t_1,t_2)}{dt_2} + kR_{v_s,v_{b'e}}(t_1,t_2) = \frac{R_{v_s,v_s}(t_1,t_2)}{cR_{s'}} \quad (2.42)$$

We know that $R_{v_s,v_s}(t_1,t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (2.42) is given as

$$R_{v_s,v_{b'e}}(t_1,t_2) = \frac{\sigma^2}{cR_{s'}} e^{k(t_1-t_2)} \quad (2.43)$$

Now, we put the value of $R_{v_s,v_{b'e}}(t_1,t_2)$ from eq. (2.43) in eq. (2.41), we obtain

$$R_{v_{b'e},v_{b'e}}(t_1,t_2) = \frac{\sigma^2}{2k(cR_s')^2}\left(e^{-k(t_1-t_2)} - e^{-k(t_1+t_2)}\right) \quad (2.44)$$

When we substitute $t_1 = t_2 = t$ in eq. (2.44), we get the second order moment of $v_{b'e}(t)$.

$$E[v_{b'e}{}^2(t)] = \frac{\sigma^2}{2k(cR_s')^2}(1 - e^{-2kt}) \quad (2.45)$$

From eqs. (2.34) and (2.45) we will have the second order moment of $v_o(t)$ that is variance of the output in this case.

$$E[v_0{}^2(t)] = (g_m R_L)^2 E[v_{b'e}{}^2(t)]$$

$$E[v_0{}^2(t)] = \frac{(g_m R_L)^2\sigma^2}{2k(cR_s')^2}(1 - e^{-2kt}) \quad (2.46)$$

2.4.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_L = 10^4\Omega$, $R_s = 5 \times 10^3\Omega$, $r_{bb'} = 100\Omega$, $r_{b'e} = 1.5 \times 10^3\Omega$, $c_e = 2pF$ $c_c = 0.8pF$, $\sigma = 0.25$, $g_m = 40mA/V$.

Fig. 2.18 represents the variation of mean with time for non-zero initial conditions ($v_{b'e}(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 2.18 that the magnitude of mean of the output has peak value 4 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 2μs (7 μs in case of single ended input BJT differential amplifier [13]). Fig. 2.19 represents the variation of variance with time. After increasing linearly with time, variance becomes constant. We also analysed the circuit for variable load resistance. Fig. 2.20 represents the variation of variance with load resistance. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 500 kHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 20 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz. The standard tool for the simulation with random input is Monte Carlo simulation. We compare our results with Monte Carlo simulation. Fig. 2.21 shows the comparison of deterministic, Monte Carlo and stochastic solution. Fig 2.21 shows that result of Monte Carlo simulation is very close to the stochastic solution and deterministic solution (The stochastic solution is very close to deterministic solution in [14] and deterministic solution is very close to stochastic and Monte Carlo solution in [20]).

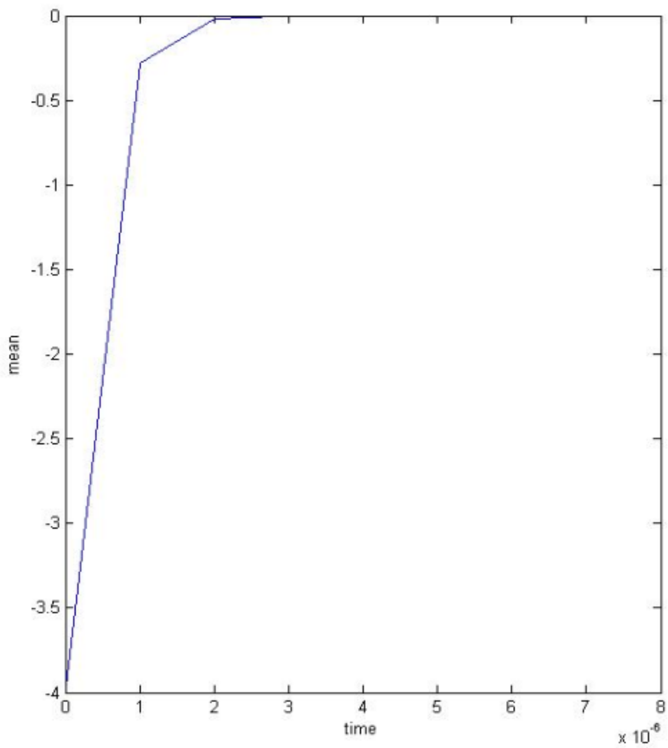

Fig. 2.18. Variation of mean with time for CE amplifier

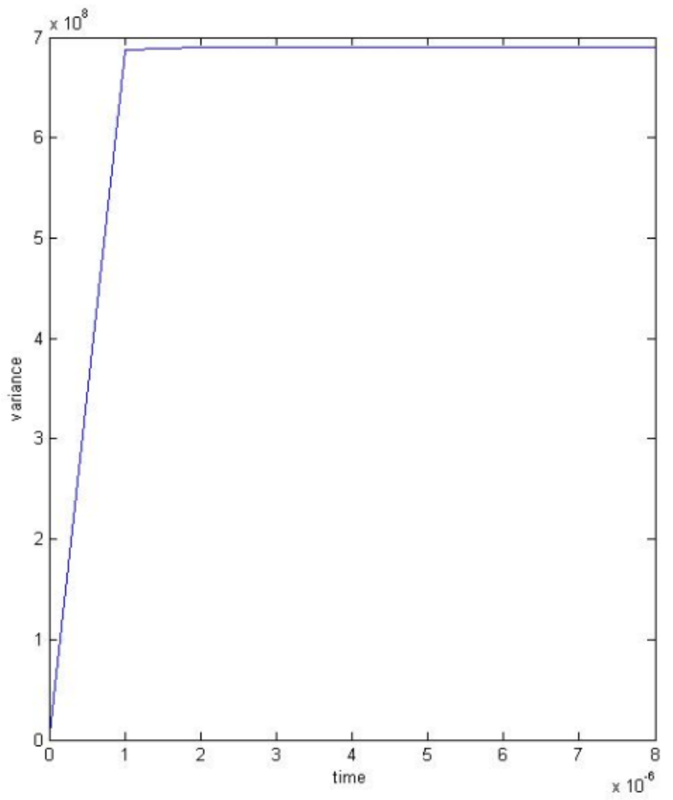

Fig. 2.19. Variation of variance with time for CE amplifier

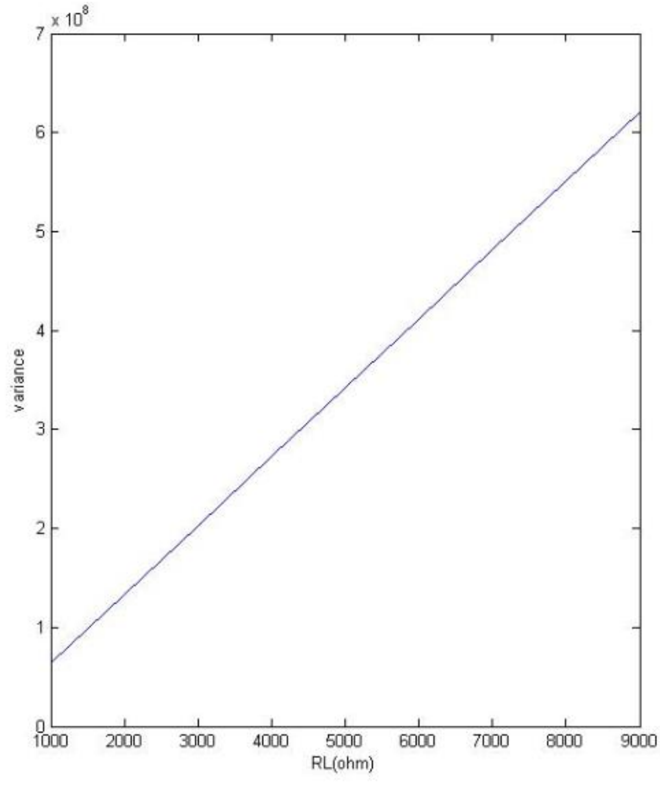

Fig. 2.20. Variation of variance with R_L for CE amplifier

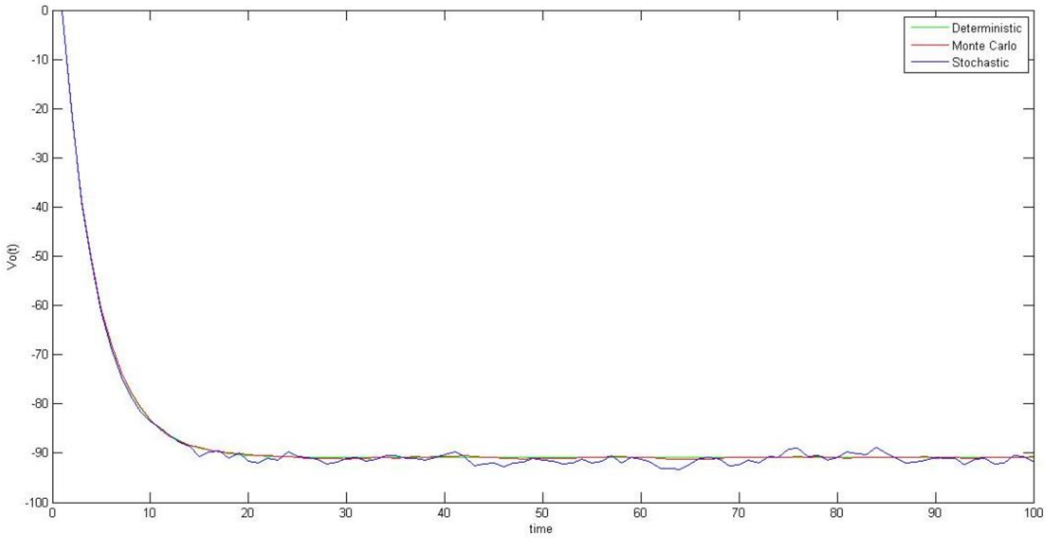

Fig. 2.21. Comparison of deterministic, stochastic and Monte Carlo simulation for CE amplifier

2.5 Conclusion

Noise analysis of different BJT amplifiers is performed. Stochastic differential equation is used to do the external noise analysis for different amplifiers. Time domain method is used to obtain the solution of stochastic differential equations. Mean and variance of the output process is determined which may be useful in design process. It has observed that noise affects the considered circuits more at high input frequencies.

CHAPTER 3

NOISE ANALYSIS OF FET AMPLIFIER CONFIGURATIONS USING SDE

3.1 INTRODUCTION

Noise analysis of different FET amplifiers is done in this chapter. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. Effect of external noise is analysed on different FET amplifiers. In this thesis, time domain method based on solving stochastic differential equation is used. To derive and compute non-Gaussian, non-stationary and nonlinear stochastic characterization of both amplitude and phase noise in an oscillator, the stochastic differential equation approach is adopted in [9]. The stochastic differential equation approach was adopted in [11] from simulation point of view for noise analysis. This method is based on linearization of stochastic differential equation about its simulated deterministic trajectory. In [12] noise analysis of sampling mixer is done. Three different sources of noise are analyzed. Conventional frequency domain method is used to analyze the external RF noise and intrinsic noise. Time domain method using stochastic differential equation is used to analyze the external local oscillator (LO) noise. In [13] noise analysis of single-ended input differential amplifier is performed using stochastic differential equation. Various statistics of output like mean and variance is obtained using stochastic differential equation. In [14] modeling of RC circuit is done to analyze the effect of external and internal noise. DC analysis of an RC circuit is performed using first order ordinary differential equation and its stochastic analogues. In [15] noise analysis of simple single stage low-pass filter (SSLPF) with the fractional-order capacitor is performed with the help of stochastic differential equation. Various solution statistics of output like mean, variance is obtained using stochastic and fractional calculus. The change in statistics with the capacitor order is investigated. The closed form solutions of the step response of fractional filter are obtained.

The noise is assumed white Gaussian noise. Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. Since external noise analysis has to be done so time domain method is used which involves solving the stochastic differential equation.

3.2 NOISE ANALYSIS OF COMMON-SOURCE AMPLIFIER USING STOCHASTIC DIFFERENTIAL EQUATION

The common-source amplifier is an important circuit in analog design. There are varieties of applications of common- source amplifier. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common- source amplifier. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although it is an ideal condition, when noise is assumed as white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Noise analysis is generally performed in frequency domain. If the circuit is linear and time invariant, this method is useful. The system can be either non-linear or time-variance if the noise analysis is done for the external noise. Therefore frequency domain method is not useful for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common- source amplifier. The autocorrelation function of the output noise and other statistics like mean and variance is obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

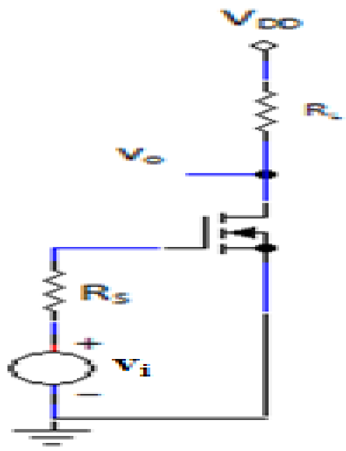

Fig.3.1. Common-Source Amplifier

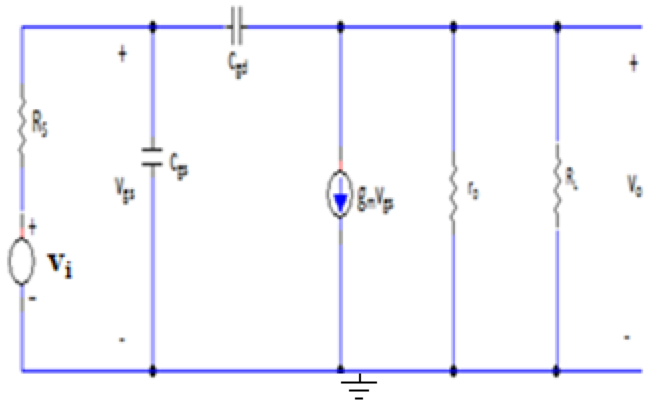

Fig.3.2. High-Frequency Equivalent Circuit of CS amplifier

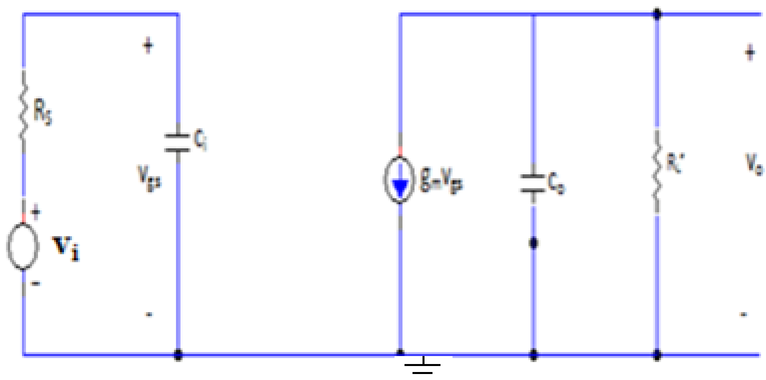

Fig.3.3. Simplified High-Frequency Equivalent Circuit of CS amplifier

3.2.1 Analysis of Noise using SDE

Common-source amplifier is considered which is shown in Fig.3.1. Fig.3.2 represents its high-frequency equivalent circuit. By Miller's theorem, c_{gd} can be transferred into input side by $c_{gd}(1+g_m R_L')$ and into output side by $c_o = c_{gd}\,(1+g_m R_L')/g_m R_L'$, which can be approximated to $c_o = c_{gd}$. So, we get the simplified equivalent circuit as shown in Fig.3.3 [29]. Now, we will analyze this circuit using SDEs. From the simplified equivalent circuit, we obtain

$$\frac{v_i(t)-v_{gs}(t)}{R_s} = c_i \frac{dv_{gs}(t)}{dt}$$

where $c_i = c_{gs} + c_{gd}(1+g_m R_L')$. Above equation can be simplified further and written as

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_i(t)}{c_i R_s} \tag{3.1}$$

where $k_1 = \frac{1}{c_i R_s}$ and

$$c_o \frac{dv_o(t)}{dt} + \frac{v_o(t)}{R_L{'}} = -g_m v_{gs}(t) \tag{3.2}$$

Consider $v_i(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_i(t) = \sigma n(t)$ in eq. (3.1)

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{\sigma n(t)}{c_i R_s}$$

$$dv_{gs}(t) + k_1 v_{gs}(t)dt = \frac{\sigma n(t)dt}{c_i R_s}$$

Then we put $n(t)dt = dW(t)$ in above equation, where $W(t)$ is considered as a Wiener process

$$dv_{gs}(t) + k_1 v_{gs}(t)dt = \frac{\sigma dW(t)}{c_i R_s} \tag{3.3}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_{gs}(t)$. We take the expectation of both side of eq. (3.3)

$$dE\left[v_{gs}(t)\right] + k_1 E\left[v_{gs}(t)\right]dt = \frac{E[\sigma dW(t)]}{c_i R_s} \tag{3.4}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (3.4) we obtain

$$\frac{dE[v_{gs}(t)]}{dt} + k_1 E[v_{gs}(t)] = 0 \tag{3.5}$$

The solution of eq. (3.5) is written as

$$E\left[v_{gs}(t)\right] = c_1 e^{-k_1 t} \tag{3.6}$$

This is the mean of $v_{gs}(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. Now, eq. (3.2) is considered to obtain the mean of the output process. Simplify and take expectation of eq. (3.2), we obtain

$$\frac{dE[v_o(t)]}{dt} + \frac{E[v_o(t)]}{c_o R_L{}'} = \frac{-g_m E[v_{gs}(t)]}{c_o} \tag{3.7}$$

The solution of this is given by

$$E[v_o(t)]e^{k_2 t} = \frac{c_2}{k_2 - k_1} e^{(k_2 - k_1)t} + c_3 \tag{3.8}$$

where $k_2 = 1/c_o R_L{}'$ and $c_2 = -g_m c_1/c_o$. c_3 is constant of integration which depends on circuit's initial conditions. Mean of output voltage will be zero if initial conditions are zero.

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eqs. (3.1) and (3.2) is rewritten

$$\frac{dv_o(t)}{dt} + k_2 v_o(t) = -\frac{g_m v_{gs}(t)}{c_o} \tag{3.9}$$

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_i(t)}{c_i R_s} \tag{3.10}$$

Eq. (3.9) is considered at time $t = t_2$ and we assume that the initial conditions for the autocorrelation of $v_o(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.9) is multiplied by $v_o(t_1)$ and then expectation is take

$$\frac{dR_{v_o,v_o}(t_1,t_2)}{dt_2} + k_2 R_{v_o,v_o}(t_1, t_2) = \frac{-g_m\, R_{v_o,v_{gs}}(t_1,t_2)}{c_0} \tag{3.11}$$

Again, eq. (3.9) is considered at time $t = t_1$ and we assume that the initial conditions for the correlation of $v_o(t)$ and $v_{gs}(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.9) is multiplied by $v_{gs}(t_2)$ and then expectation is taken

$$\frac{dR_{v_o,v_{gs}}(t_1,t_2)}{dt_1} + k_2 R_{v_o,v_{gs}}(t_1,t_2) = \frac{-g_m R_{v_{gs},v_{gs}}(t_1,t_2)}{c_0} \quad (3.12)$$

Eq. (3.10) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_{gs}(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.10) is multiplied by $v_{gs}(t_2)$ and then expectation is taken

$$\frac{dR_{v_{gs},v_{gs}}(t_1,t_2)}{dt_1} + k_1 R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{R_{v_i,v_{gs}}(t_1,t_2)}{c_i R_S} \quad (3.13)$$

Again, eq. (3.10) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_i(t)$ and $v_{gs}(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.10) is multiplied by $v_i(t_1)$ and then expectation is taken

$$\frac{dR_{v_i,v_{gs}}(t_1,t_2)}{dt_2} + k_1 R_{v_i,v_{gs}}(t_1,t_2) = \frac{R_{v_i,v_i}(t_1,t_2)}{c_i R_S} \quad (3.14)$$

Now, we want to have solutions of differential eqs. (3.11), (3.12), (3.13) and (3.14) to find out the value of $R_{v_o,v_o}(t_1,t_2)$. We know that $R_{v_i,v_i}(t_1,t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (3.14) is given as

$$R_{v_i,v_{gs}}(t_1,t_2) = \frac{\sigma^2}{c_i R_S} e^{k_1(t_1-t_2)} \quad (3.15)$$

Now, we put the value of $R_{v_i,v_{gs}}(t_1,t_2)$ from eq. (3.15) in eq. (3.13), we obtain

$$R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{\sigma^2}{2k_1(R_S c_i)^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \quad (3.16)$$

Now, we put the value of $R_{v_{gs},v_{gs}}(t_1,t_2)$ from eq. (3.16) in eq. (3.12), we obtain

$$R_{v_o,v_{gs}}(t_1,t_2) = \frac{k_3}{\frac{1}{R_L'} - k_1 c_o}\left(e^{\left(\frac{t_2-t_1}{c_O R_L'}\right)} - e^{\left(k_1 t_2 - \frac{t_1}{c_O R_L'}\right)} - e^{\left(-2k_1 t_2 + \frac{t_2-t_1}{c_O R_L'}\right)} + e^{\left(-k_1 t_2 - \frac{t_1}{c_O R_L'}\right)}\right) \quad (3.17)$$

where $k_3 = \frac{-g_m \sigma^2}{2k_1 (R_s c_i)^2}$. Now, we put the value of $R_{v_o, v_{gs}}(t_1, t_2)$ from eq. (3.17) in eq. (3.11), we obtain

$$R_{v_o, v_o}(t_1, t_2) = \frac{-g_m k_3}{\frac{1}{R_L'} - k_1 c_o} \left(\left(e^{\frac{t_2 - t_1}{c_o R_L'}} - e^{\frac{-t_1 - t_2}{c_o R_L'}} \right) \frac{c_o R_L'}{2} + \frac{\left(e^{-2k_1 t_2 + \frac{t_2 - t_1}{c_o R_L'}} - 2 e^{k_1 t_2 - \frac{t_1}{c_o R_L'}} + e^{\frac{-t_1 - t_2}{c_o R_L'}} \right)}{2\left(k_1 + \frac{1}{c_o R_L'} \right)} - \frac{\left(e^{-k_1 t_2 - \frac{t_1}{c_o R_L'}} - e^{\frac{-t_1 - t_2}{c_o R_L'}} \right)}{\frac{1}{c_o R_L'} - k_1} \right) \quad (3.18)$$

When we substitute $t_1 = t_2 = t$ in eq. (3.18), we get the second order moment of $v_o(t)$, that is variance of the output in this case.

$$E[v_o^2(t)] = \frac{-g_m k_3}{\frac{1}{R_L'} - k_1 c_o} \left(\left(1 - e^{\frac{-2t}{c_o R_L'}} \right) \frac{c_o R_L'}{2} + \frac{\left(e^{-2k_1 t} - 2 e^{\left(k_1 - \frac{1}{c_o R_L'} \right) t} + e^{\frac{-2t}{c_o R_L'}} \right)}{2\left(k_1 + \frac{1}{c_o R_L'} \right)} - \frac{\left(e^{\left(-k_1 - \frac{1}{c_o R_L'} \right) t} - e^{\frac{-2t}{c_o R_L'}} \right)}{\frac{1}{c_o R_L'} - k_1} \right) \quad (3.19)$$

3.2.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_L = 10k\Omega$, $R_S = 5k\Omega$, $r_o = 44k\Omega$, $\sigma = 0.25$, $c_{gs} = 3pF$, $c_{gd} = 2.8pF$, $g_m = 0.0016A/V$.

Fig.3.4 represents the variation of mean of output with time for non-zero initial conditions ($v_{gs}(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 3.4 that the magnitude of mean of the output has peak value 0.13 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 1.5μs (7μs in case of single ended input BJT differential amplifier [13]). Fig.3.5 represents the variation of variance of output with time. We can observe that the variance attain a constant value of approximately 4×10^{-5} after 1μs. The variance has the maximum value of approximately 5.1×10^{-5}. We also analysed the circuit for variable load resistance. Fig.3.6 represents the variation of mean with load resistance. Fig.3.7 represents the variation of variance with load resistance. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 666.67 kHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 15 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz.

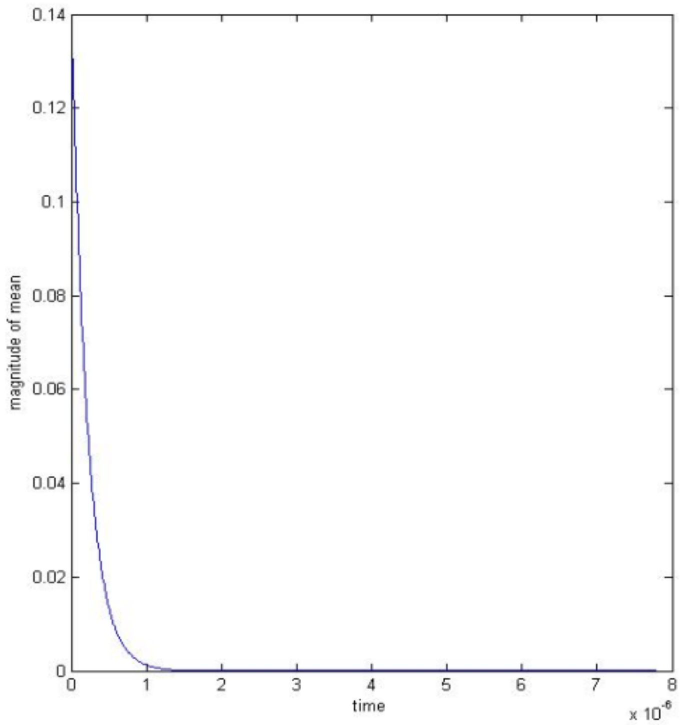

Fig.3.4. Variation of mean with time for CS amplifier

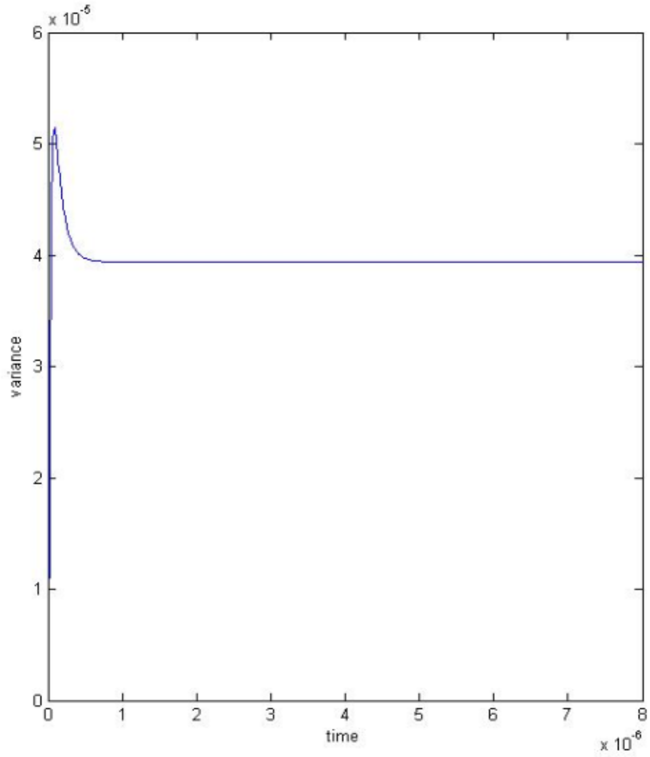

Fig.3.5. Variation of variance with time for CS amplifier

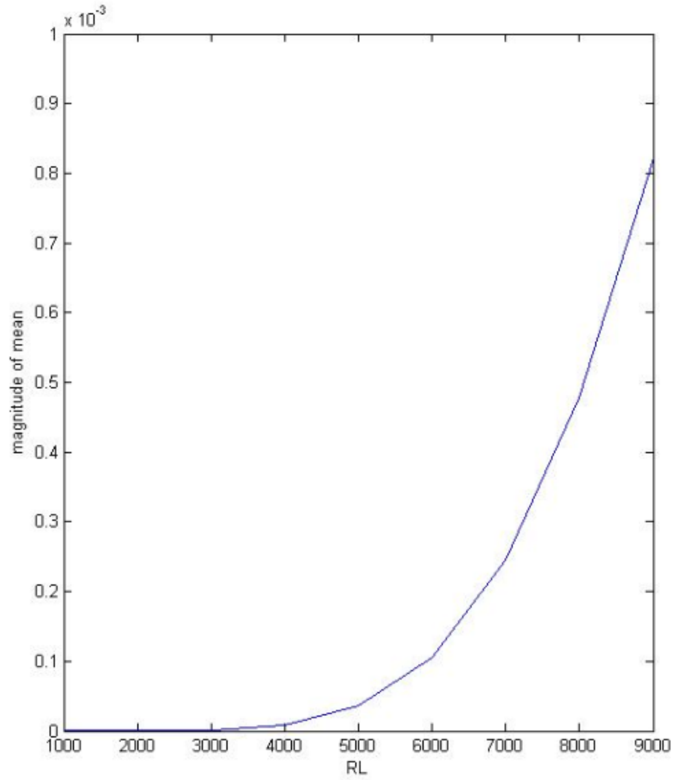

Fig.3.6 Variation of mean with load resistance for CS amplifier

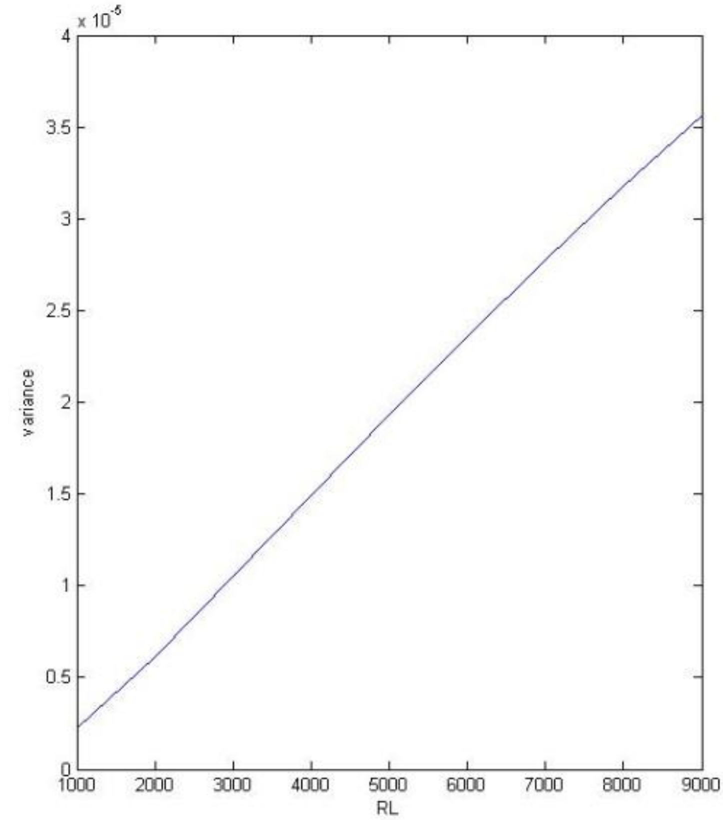

Fig.3.7 Variation of variance with load resistance for CS amplifier

3.3 STOCHASTIC DIFFERENTIAL EQUATION NOISE ANALYSIS OF COMMON-SOURCE AMPLIFIER WITH CAPACITIVE LOAD

The common-source amplifier is an important circuit in analog design. There are varieties of applications of common- source amplifier. The noise which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common-source amplifier with capacitive load. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although It is an ideal condition, when noise is assumed as white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common-source amplifier with capacitive load. The autocorrelation function of the output noise and other statistics like mean and variance are obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

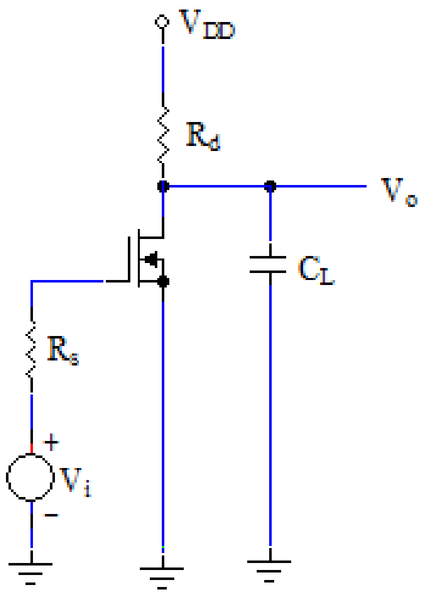

Fig.3.8. Common-Source Amplifier with capacitive load

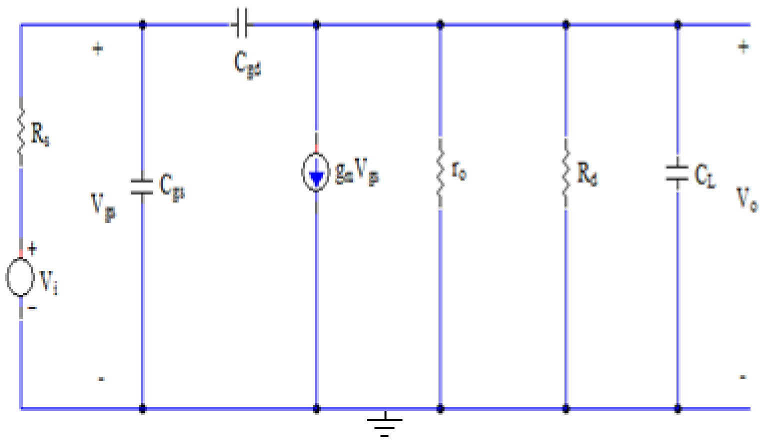

Fig.3.9. High-Frequency Equivalent Circuit of CS amplifier (capacitive load)

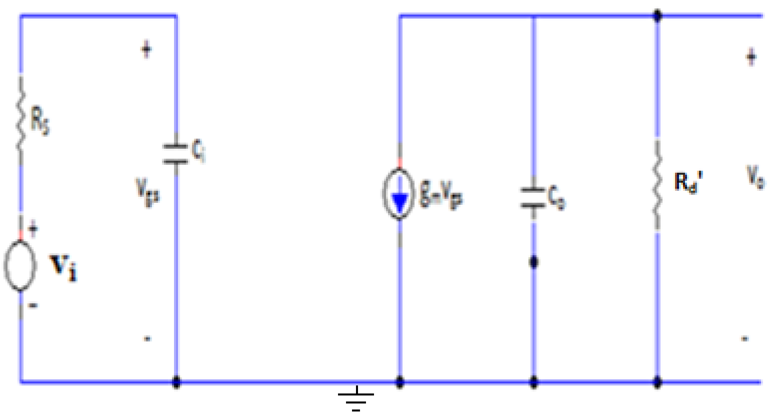

Fig.3.10. Simplified High-Frequency Equivalent Circuit of CS amplifier (capacitive load)

3.3.1 Analysis of Noise using SDE

Common-source amplifier with capacitive load is considered which is shown in Fig.3.8. Fig.3.9 represents its high-frequency equivalent circuit. By Miller's theorem, c_{gd} can be transferred into input side by $c_i' = c_{gd}(1 + g_m R_d')$ and into output side by $c_o' = c_{gd}\,(1 + g_m R_d')/g_m R_d'$, which can be approximated to $c_o' = c_{gd}$. So, we get the simplified equivalent circuit as shown in Fig.3.10. Now, we will analyze this circuit using SDEs. From the simplified equivalent circuit, we obtain

$$\frac{v_i(t) - v_{gs}(t)}{R_s} = c_i \frac{dv_{gs}(t)}{dt}$$

where $c_i = c_{gs} + c_{gd}(1 + g_m R_d{}')$ and $R_d{}' = (r_o R_d)/r_o R_d$. Above equation can be simplified further and written as

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_i(t)}{c_i R_s} \tag{3.20}$$

where $k_1 = \frac{1}{c_i R_s}$

$$c_o \frac{dv_o(t)}{dt} + \frac{v_o(t)}{R_d{}'} = -g_m v_{gs}(t) \tag{3.21}$$

where $c_0 = (c_\mathrm{L} + c_o{}')$

Consider $v_i(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_i(t) = \sigma n(t)$ in eq. (3.20)

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{\sigma n(t)}{c_i R_s}$$

$$dv_{gs}(t) + k_1 v_{gs}(t)dt = \frac{\sigma n(t)dt}{c_i R_s}$$

Then we put $n(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_{gs}(t) + k_1 v_{gs}(t)dt = \frac{\sigma dW(t)}{c_i R_s} \tag{3.22}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_{gs}(t)$. We take the expectation of both side of eq. (3.22)

$$dE\left[v_{gs}(t)\right] + k_1 E\left[v_{gs}(t)\right]dt = \frac{E[\sigma dW(t)]}{c_i R_s} \tag{3.23}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (3.23) we obtain

$$\frac{dE[v_{gs}(t)]}{dt} + k_1 E[v_{gs}(t)] = 0 \tag{3.24}$$

The solution of eq. (3.24) is written as

$$E\left[v_{gs}(t)\right] = c_1 e^{-k_1 t} \tag{3.25}$$

This is the mean of $v_{gs}(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. Now, eq. (3.21) is considered to obtain the mean of the output process. Simplify and take expectation of eq. (3.21), we obtain

$$\frac{dE[v_o(t)]}{dt} + \frac{E[v_o(t)]}{c_o R_d{}'} = \frac{-g_m E[v_{gs}(t)]}{c_o} \tag{3.26}$$

The solution of this is given by

$$E[v_o(t)]e^{k_2 t} = \frac{c_2}{k_2 - k_1} e^{(k_2 - k_1)t} + c_3 \tag{3.27}$$

where $k_2 = 1/c_o R_d{}'$ and $c_2 = -g_m c_1/c_o$. c_3 is constant of integration which depends on circuit's initial conditions. Mean of output voltage will be zero if initial conditions are zero.

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eqs. (3.20) and (3.21) is rewritten

$$\frac{dv_o(t)}{dt} + k_2 v_o(t) = -\frac{g_m v_{gs}(t)}{c_o} \tag{3.28}$$

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_i(t)}{c_i R_s} \tag{3.29}$$

Eq. (3.28) is considered at time $t = t_2$ and we assume that the initial conditions for the autocorrelation of $v_o(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.28) is multiplied by $v_o(t_1)$ and then expectation is taken

$$\frac{dR_{v_o,v_o}(t_1,t_2)}{dt_2} + k_2 R_{v_o,v_o}(t_1,t_2) = \frac{-g_m\, R_{v_o,v_{gs}}(t_1,t_2)}{c_0} \tag{3.30}$$

Again, eq. (3.28) is considered at time $t = t_1$ and we assume that the initial conditions for the correlation of $v_o(t)$ and $v_{gs}(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.28) is multiplied by $v_{gs}(t_2)$ and then expectation is taken

$$\frac{dR_{v_o,v_{gs}}(t_1,t_2)}{dt_1} + k_2 R_{v_o,v_{gs}}(t_1,t_2) = \frac{-g_m\, R_{v_{gs},v_{gs}}(t_1,t_2)}{c_0} \tag{3.31}$$

Eq. (3.29) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_{gs}(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.29) is multiplied by $v_{gs}(t_2)$ and then expectation is taken

$$\frac{dR_{v_{gs},v_{gs}}(t_1,t_2)}{dt_1} + k_1 R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{R_{v_i,v_{gs}}(t_1,t_2)}{c_i R_S} \tag{3.32}$$

Again, eq. (3.29) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_i(t)$ and $v_{gs}(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.29) is multiplied by $v_i(t_1)$ and then expectation is taken

$$\frac{dR_{v_i,v_{gs}}(t_1,t_2)}{dt_2} + k_1 R_{v_i,v_{gs}}(t_1,t_2) = \frac{R_{v_i,v_i}(t_1,t_2)}{c_i R_S} \tag{3.33}$$

Now, we want to have solutions of differential eqs. (3.30), (3.31), (3.32) and (3.33) to find out the value of $R_{v_o,v_o}(t_1,t_2)$. We know that $R_{v_i,v_i}(t_1,t_2) = \sigma^2 \delta(t_1 - t_2)$, the solution of eq. (3.33) is given as

$$R_{v_i,v_{gs}}(t_1,t_2) = \frac{\sigma^2}{c_i R_S} e^{k_1(t_1 - t_2)} \tag{3.34}$$

Now, we put the value of $R_{v_i,v_{gs}}(t_1,t_2)$ from eq. (3.34) in eq. (3.32), we obtain

$$R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{\sigma^2}{2k_1(R_s c_i)^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \tag{3.35}$$

Now, we put the value of $R_{v_{gs},v_{gs}}(t_1,t_2)$ from eq. (3.35) in eq. (3.31), we obtain

$$R_{v_o,v_{gs}}(t_1,t_2) = \frac{k_3}{\frac{1}{R_d'} - k_1 c_o}\left(e^{\left(\frac{t_2-t_1}{c_O R_d'}\right)} - e^{\left(k_1 t_2 - \frac{t_1}{c_O R_d'}\right)} - e^{\left(-2k_1 t_2 + \frac{t_2 - t_1}{c_O R_d'}\right)} + e^{\left(-k_1 t_2 - \frac{t_1}{c_O R_d'}\right)}\right) \tag{3.36}$$

where $k_3 = \frac{-g_m \sigma^2}{2k_1(R_s c_i)^2}$. Now, we put the value of $R_{v_o,v_{gs}}(t_1,t_2)$ from eq. (3.36) in eq. (3.30), we obtain

$R_{v_o,v_o}(t_1,t_2) =$

$$\frac{-g_m k_3}{\frac{1}{R_d{}'}-k_1 c_o}\left(\left(e^{\frac{t_2-t_1}{c_o R_d{}'}} - e^{\frac{-t_1-t_2}{c_o R_d{}'}}\right)\frac{c_o R_d{}'}{2} + \frac{\left(e^{-2k_1 t_2+\frac{t_2-t_1}{c_o R_d{}'}}-2e^{k_1 t_2-\frac{t_1}{c_o R_d{}'}}+e^{\frac{-t_1-t_2}{c_o R_d{}'}}\right)}{2\left(k_1+\frac{1}{c_o R_d{}'}\right)} - \frac{\left(e^{-k_1 t_2-\frac{t_1}{c_o R_d{}'}}-e^{\frac{-t_1-t_2}{c_o R_d{}'}}\right)}{\frac{1}{c_o R_d{}'}-k_1}\right) \tag{3.37}$$

When we substitute $t_1 = t_2 = t$ in eq. (3.37), we get the second order moment of $v_o(t)$ that is variance of the output in this case.

$$E[v_o^2(t)] = \frac{-g_m k_3}{\frac{1}{R_d{}'}-k_1 c_o}\left(\left(1 - e^{\frac{-2t}{c_o R_d{}'}}\right)\frac{c_o R_d{}'}{2} + \frac{\left(e^{-2k_1 t}-2e^{\left(k_1-\frac{1}{c_o R_d{}'}\right)t}+e^{\frac{-2t}{c_o R_d{}'}}\right)}{2\left(k_1+\frac{1}{c_o R_d{}'}\right)} - \frac{\left(e^{\left(-k_1-\frac{1}{c_o R_d{}'}\right)t}-e^{\frac{-2t}{c_o R_d{}'}}\right)}{\frac{1}{c_o R_d{}'}-k_1}\right) \tag{3.38}$$

3.3.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_d = 10k\Omega$, $R_S = 5k\Omega$, $r_o = 44k\Omega$, $\sigma = 0.25,$ $c_{gs} = 3pF,$ $c_{gd} = 2.8pF, c_L = 2pF$ $g_m = 0.0016A/V.$

Fig. 3.11 represents the variation of mean of output with time for non-zero initial conditions ($v_{gs}(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 3.11 that the magnitude of mean of the output has peak value 0.14 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 1.5μs (7μs in case of single ended input BJT differential amplifier [13]). Fig.3.12 represents the variation of variance of output with time. We can observe that the variance attain a constant value of approximately 7.4×10^{-5} after 1μs. The variance has the maximum value of approximately 8.2×10^{-5}. Fig.3.13 represents the variation of variance with C_L. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 666.67 kHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 15 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz.

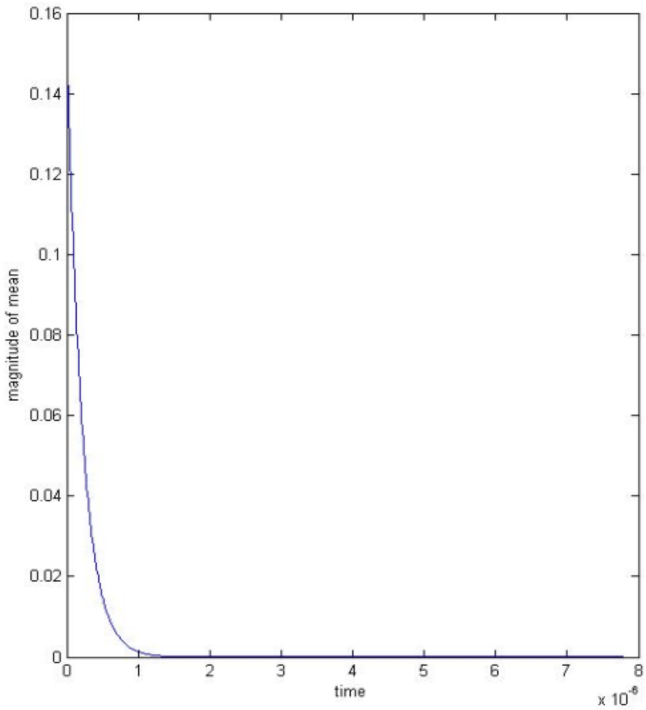

Fig.3.11. Variation of mean with time for CS amplifier (capacitive load)

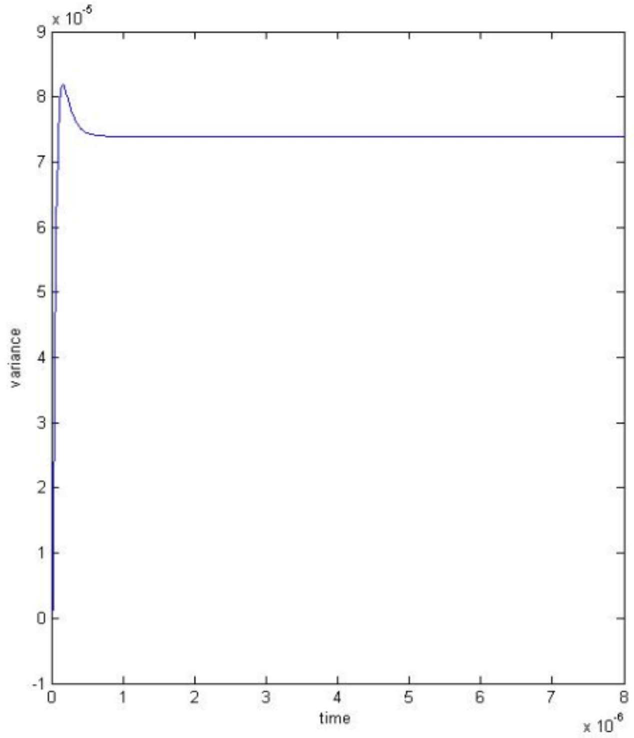

Fig.3.12. Variation of variance with time for CS amplifier (capacitive load)

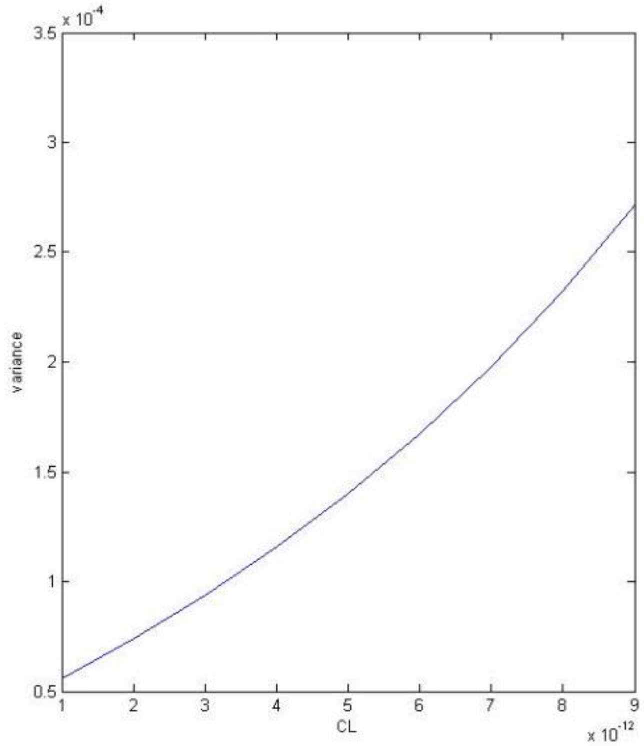

Fig.3.13. Variation of variance with C_L for CS amplifier (capacitive load)

3.4 NOISE ANALYSIS OF COMMON-DRAIN AMPLIFIER USING STOCHASTIC DIFFERENTIAL EQUATION

The common-drain amplifier is an important circuit in analog design. There are varieties of applications of common- drain amplifier. The noise which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common- drain amplifier. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although It is an ideal condition, when noise is assumed as white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common- drain amplifier. The autocorrelation function of the output noise and other statistics like mean and variance are obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

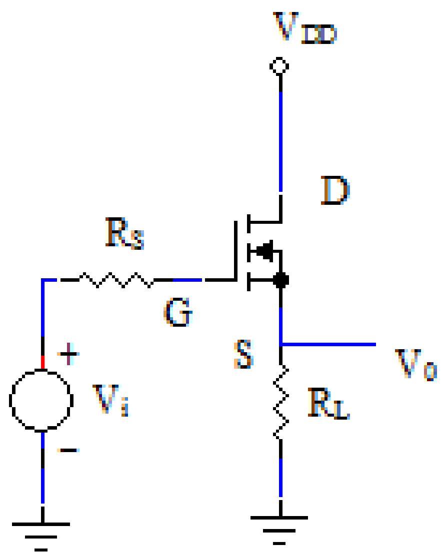

Fig.3.14. Common-Drain Amplifier

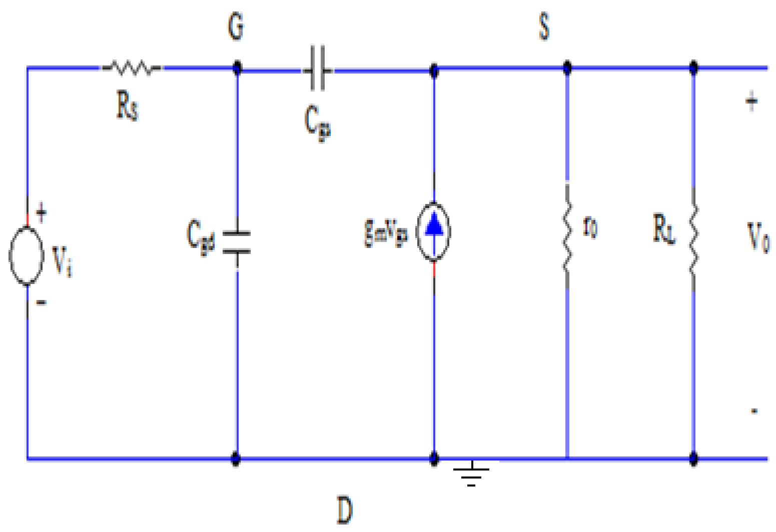

Fig.3.15. High-Frequency Equivalent Circuit of CD amplifier

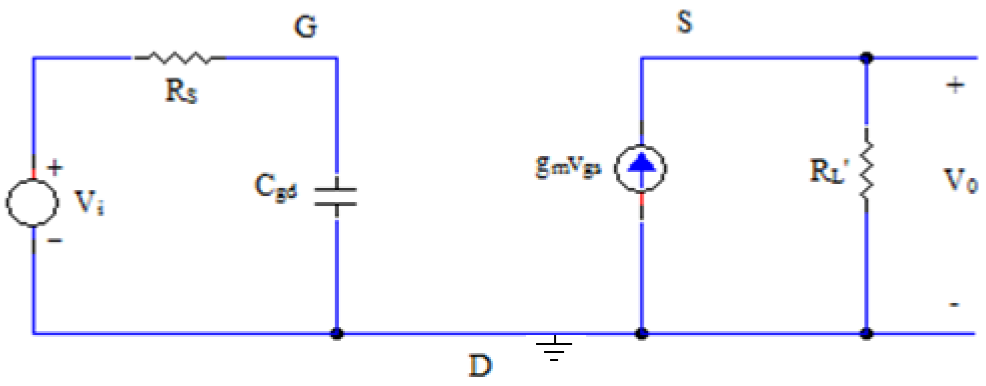

Fig.3.16. Simplify High-Frequency Equivalent Circuit of CD amplifier

3.4.1 Analysis of Noise using SDE

Common-drain amplifier is considered which is shown Fig.3.14. Fig.3.15 represents its high-frequency equivalent circuit. By Miller's theorem, c_{gs} can be transferred into input side by $c_{gs}(1-k)$ and into output side by $c_{gs}\,(k-1)/k$. As the gain (k) for the common-drain amplifier is approximately 1, the value of $(1-k)$ is close to zero. So this approximation leads us to obtain the simplified high-frequency equivalent circuit as shown in Fig.3.16. Now, we will analyse this circuit using SDEs. From the simplified equivalent circuit, we obtain

$$\frac{v_i(t)-v_g(t)}{R_s} = c_{gd}\frac{dv_g(t)}{dt} \tag{3.39}$$

Eq. (3.39) can be simplified further and written as

$$\frac{dv_g(t)}{dt} + k_1 v_g(t) = \frac{v_i(t)}{c_{gd}R_s} \tag{3.40}$$

where $k_1 = \frac{1}{c_{gd}R_s}$

$$v_0(t) = g_m R_L{}' v_{gs}(t)$$

$$v_0(t) = \frac{g_m R_{L'}}{1+g_m R_{L'}} v_g(t) \tag{3.41}$$

where $v_s(t) = v_0(t)$. Consider $v_i(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_i(t) = \sigma n(t)$ in eq. (3.40)

$$\frac{dv_g(t)}{dt} + k_1 v_g(t) = \frac{\sigma n(t)}{c_{gd}R_s}$$

$$dv_g(t) + k_1 v_g(t)dt = \frac{\sigma n(t)dt}{c_{gd}R_s}$$

Then we put $n(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_g(t) + k_1 v_g(t)dt = \frac{\sigma dW(t)}{c_{gd}R_s} \tag{3.42}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_g(t)$. We take the expectation of both side of eq. (3.42)

$$dE\left[v_g(t)\right] + k_1 E\left[v_g(t)\right]dt = \frac{E[\sigma dW(t)]}{c_{gd}R_s} \tag{3.43}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (3.43) we obtain

$$\frac{dE[v_g(t)]}{dt} + k_1 E[v_g(t)] = 0 \tag{3.44}$$

The solution of eq. (3.44) is written as

$$E\left[v_g(t)\right] = c_1 e^{-k_1 t} \tag{3.45}$$

This is the mean of $v_g(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. From eqs. (3.41) and (3.45) we can obtain the mean of the output

$$E[v_0(t)] = \frac{g_m {R_L}'}{1 + g_m {R_L}'} E\left[v_g(t)\right]$$

$$E[v_0(t)] = \frac{g_m R_{L'}}{1+g_m R_{L'}} c_1 e^{-k_1 t} \tag{3.46}$$

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eq. (3.40) is rewritten

$$\frac{dv_g(t)}{dt} + k_1 v_g(t) = \frac{v_i(t)}{c_{gd}R_s} \tag{3.47}$$

Eq. (3.47) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_g(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.47) is multiplied by $v_g(t_2)$ and then expectation is taken

$$\frac{dR_{v_g,v_g}(t_1,t_2)}{dt_1} + k_1 R_{v_g,v_g}(t_1, t_2) = \frac{R_{v_i,v_g}(t_1,t_2)}{c_{gd}R_s} \tag{3.48}$$

Again, eq. (3.47) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_i(t)$ and $v_g(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.47) is multiplied by $v_i(t_1)$ and then expectation is taken

$$\frac{dR_{v_i,v_g}(t_1,t_2)}{dt_2} + k_1 R_{v_i,v_g}(t_1, t_2) = \frac{R_{v_i,v_i}(t_1,t_2)}{c_{gd}R_s} \tag{3.49}$$

We know that $R_{v_i,v_i}(t_1, t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (3.49) is given as

$$R_{v_i,v_g}(t_1, t_2) = \frac{\sigma^2}{c_{gd}R_s} e^{k_1(t_1-t_2)} \tag{3.50}$$

Now, we put the value of $R_{v_i,v_g}(t_1, t_2)$ from eq. (3.50) in eq. (3.48), we obtain

$$R_{v_g,v_g}(t_1, t_2) = \frac{\sigma^2}{2k_1\left(c_{gd}R_s\right)^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \tag{3.51}$$

When we substitute $t_1 = t_2 = t$ in eq. (3.51), we get the second order moment of $v_g(t)$.

$$E\left[v_g^{\,2}(t)\right] = \frac{\sigma^2}{2k_1\left(c_{gd}R_s\right)^2}\left(1 - e^{-2k_1 t}\right) \tag{3.52}$$

From eqs. (3.41) and (3.52) we will have the second order moment of $v_o(t)$ that is variance of the output in this case.

$$E[v_0^{\,2}(t)] = \frac{(g_m R_L')^2}{(1 + g_m R_L')^2} E\left[v_g^{\,2}(t)\right]$$

$$E[v_0^{\,2}(t)] = \frac{(g_m R_L')^2\sigma^2}{(1+g_m R_L')^2 2k_1\left(c_{gd}R_s\right)^2}\left(1 - e^{-2k_1 t}\right) \tag{3.53}$$

3.4.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_L = 10k\Omega$, $R_S = 5k\Omega$, $r_o = 44k\Omega$, $\sigma = 0.25$, $c_{gd} = 2.8pF$, $g_m = 0.0016A/V$.

Fig. 3.17 represents the variation of mean with time for non-zero initial conditions ($v_g\,(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 3.17 that the magnitude of mean of the output has peak value 7.5×10^{-3} volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 1μs (7 μs in case of single ended input BJT differential amplifier [13]). Fig.3.18 represents the variation of variance with time. After increasing linearly with time, variance becomes constant. We also analysed the circuit for variable load resistance. Fig.3.19 represents the variation of variance with load resistance. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 1MHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 10 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz. The standard tool for the simulation with random input is Monte Carlo simulation. We compare our results with Monte Carlo simulation. Fig. 3.20 shows the comparison of deterministic, Monte Carlo and stochastic solution. Fig 3.20 shows that result of Monte Carlo simulation is very close to the stochastic solution and deterministic solution (The stochastic solution is very close to deterministic solution in [14] and deterministic solution is very close to stochastic and Monte Carlo solution in [20]).

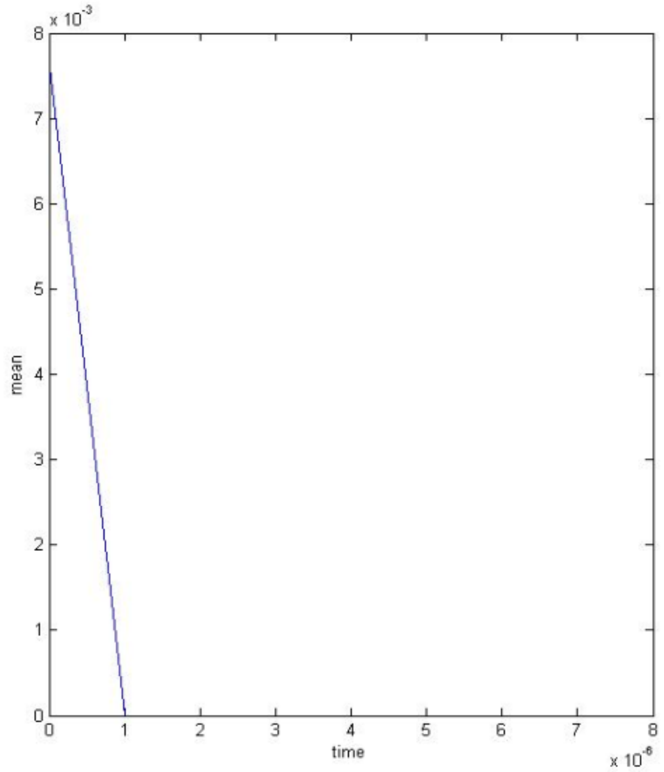

Fig.3.17. Variation of mean with time for CD amplifier

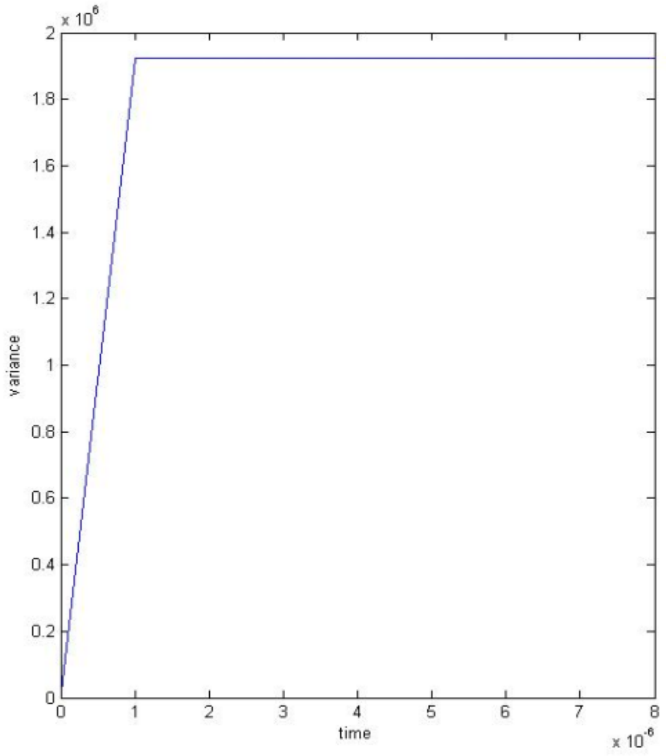

Fig.3.18. Variation of variance with time for CD amplifier

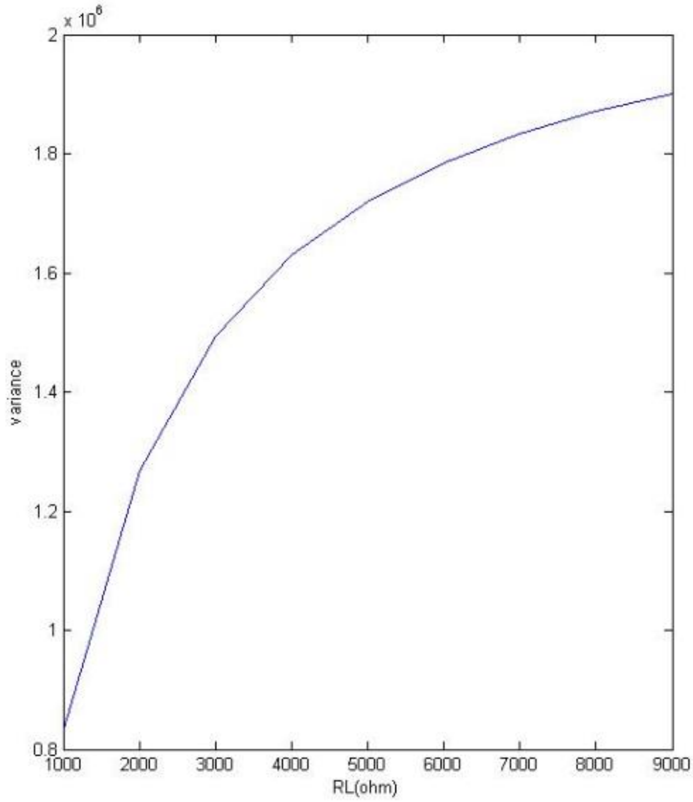

Fig.3.19. Variation of variance with R_L for CD amplifier

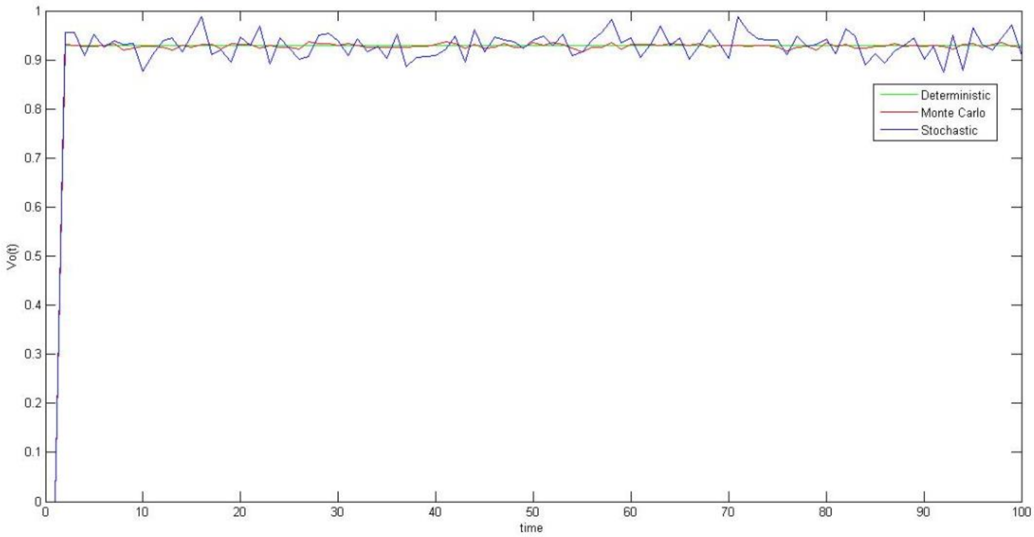

Fig.3.20. Comparison of deterministic, stochastic and Monte Carlo simulation for CD amplifier

3.5 NOISE ANALYSIS OF COMMON-GATE AMPLIFIER USING STOCHASTIC DIFFERENTIAL EQUATION

The common-gate amplifier is an important circuit in analog design. There are varieties of applications of common- gate amplifier. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common-gate amplifier. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although It is an ideal condition, when noise is assumed as white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common-gate amplifier. The autocorrelation function of the output noise and other statistics like mean and variance are obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

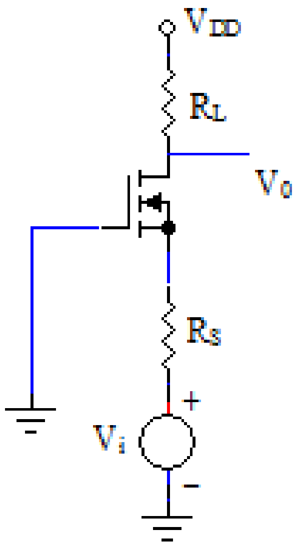

Fig.3.21. Common-Gate Amplifier

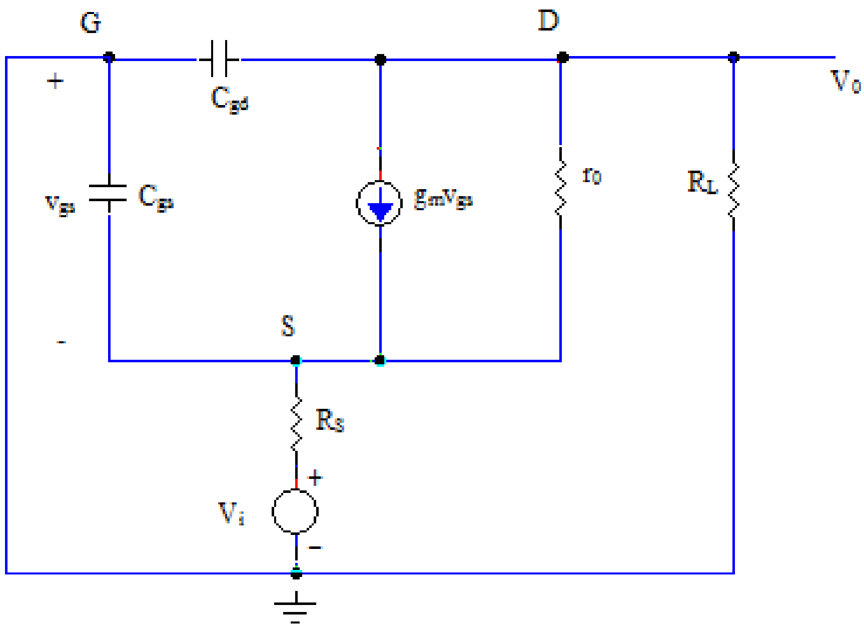

Fig.3.22. High-Frequency Equivalent Circuit of CG amplifier

3.5.1 Analysis of Noise using SDE

We consider common-gate amplifier which is shown Fig.3.21. Its high-frequency equivalent circuit is given in Fig.3.22. Now we will analyse this circuit using SDEs. If we assume that the value of R_s is very small, then $v_s = v_i$. From the circuit, we get

$$c_{gd}\frac{dv_0}{dt}+\frac{v_0}{R_L}+\frac{v_0-v_i}{r_0}=-g_m v_{gs} \qquad (3.54)$$

Eq. (3.54) can be simplified further and written as

$$\frac{dv_0(t)}{dt}+kv_0(t)=gv_i \qquad (3.55)$$

where $k=\frac{1}{c_{gd}}(\frac{1}{R_L}+\frac{1}{r_0})$ and $g=(g_m+\frac{1}{r_0})\frac{1}{c_{gd}}$. Consider $v_i(t)=\sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_i(t)=\sigma n(t)$ eq. in (3.55)

$$\frac{dv_0(t)}{dt}+kv_0(t)=g\sigma n(t)$$

$$dv_0(t)+kv_0(t)dt=g\sigma n(t)dt$$

Then we put $n(t)dt=dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_0(t)+kv_0(t)dt=g\sigma dW(t) \qquad (3.56)$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we take the expectation of both side of eq. (3.56)

$$dE[v_0(t)]+kE[v_0(t)]dt=E[g\sigma dW(t)] \qquad (3.57)$$

For the Wiener process, $E[\sigma dW(t)]=0$, so from eq. (3.57) we obtain

$$\frac{dE[v_0(t)]}{dt}+kE[v_0(t)]=0 \qquad (3.58)$$

The solution of eq. (3.58) is written as

$$E[v_0(t)]=c_1e^{-kt} \qquad (3.59)$$

This is the mean of output process. Where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions.

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eq. (3.55) is rewritten

$$\frac{dv_0(t)}{dt} + kv_0(t) = gv_i \tag{3.60}$$

Now, consider eq. (3.60) $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_o(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.60) is multiplied by $v_o(t_2)$ and then expectation is taken

$$\frac{dR_{v_0,v_0}(t_1,t_2)}{dt_1} + kR_{v_0,v_0}(t_1, t_2) = gR_{v_i,v_0}(t_1, t_2) \tag{3.61}$$

Again, consider eq. (3.60) at $t = t_2$ and we assume that the initial conditions for the correlation of $v_i(t)$ and $v_o(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.60) is multiplied by $v_i(t_1)$ and then expectation is taken

$$\frac{dR_{v_i,v_0}(t_1,t_2)}{dt_2} + kR_{v_i,v_0}(t_1, t_2) = gR_{v_i,v_i}(t_1, t_2) \tag{3.62}$$

We know that $R_{v_i,v_i}(t_1, t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (3.62) is given as

$$R_{v_i,v_0}(t_1, t_2) = g\sigma^2 e^{k(t_1-t_2)} \tag{3.63}$$

Now, we put the value of $R_{v_i,v_o}(t_1, t_2)$ from eq. (3.63) in eq. (3.61), we obtain

$$R_{v_0,v_0}(t_1, t_2) = \frac{g^2\sigma^2}{2k}\left(e^{-k(t_1-t_2)} - e^{-k(t_1+t_2)}\right) \tag{3.64}$$

When we substitute $t_1 = t_2 = t$ in eq. (3.64) we will have the second moment of the $v_o(t)$ that is variance of the output in this case

$$E[{v_0}^2(t)] = \frac{g^2\sigma^2}{2k}(1 - e^{-2kt}) \tag{3.65}$$

3.5.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $R_L = 10k\Omega$, $r_o = 44k\Omega$, $\sigma = 0.25$, $c_{gd} = 2.8pF$, $g_m = 0.0016A/V$.

Fig.3.23 represents the variation of mean of output with time for non-zero initial conditions ($v_o\,(0) = 0.01V$). The mean will be zero for zero initial conditions. It has observed from Fig. 3.23 that the magnitude of mean of the output has peak value 0.01 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 1μs (7 μs in case of single ended input BJT differential amplifier [13]). Fig.3.24 represents the variation of variance of output with time. After increasing linearly with time, variance becomes constant. We also analysed the circuit for variable load resistance. Fig.3.25 represents the variation of variance with load resistance. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 1MHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 10 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz. The standard tool for the simulation with random input is Monte Carlo simulation. We compare our results with Monte Carlo simulation. Fig. 3.26 shows the comparison of deterministic, Monte Carlo and stochastic solution. Fig 3.26 shows that result of Monte Carlo simulation is very close to the stochastic solution and deterministic solution (The stochastic solution is very close to deterministic solution in [14] and deterministic solution is very close to stochastic and Monte Carlo solution in [20]).

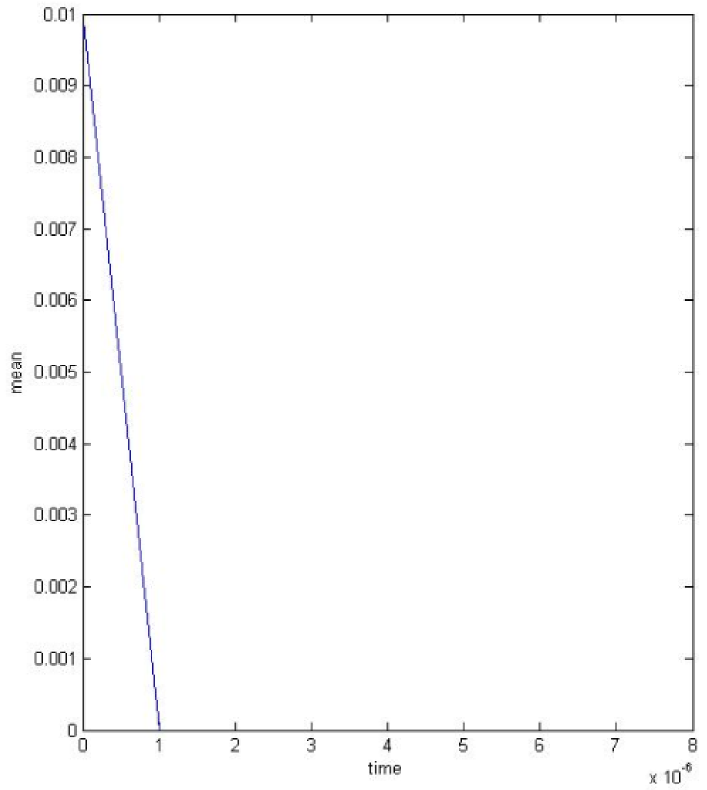

Fig.3.23. Variation of mean with time for CG amplifier

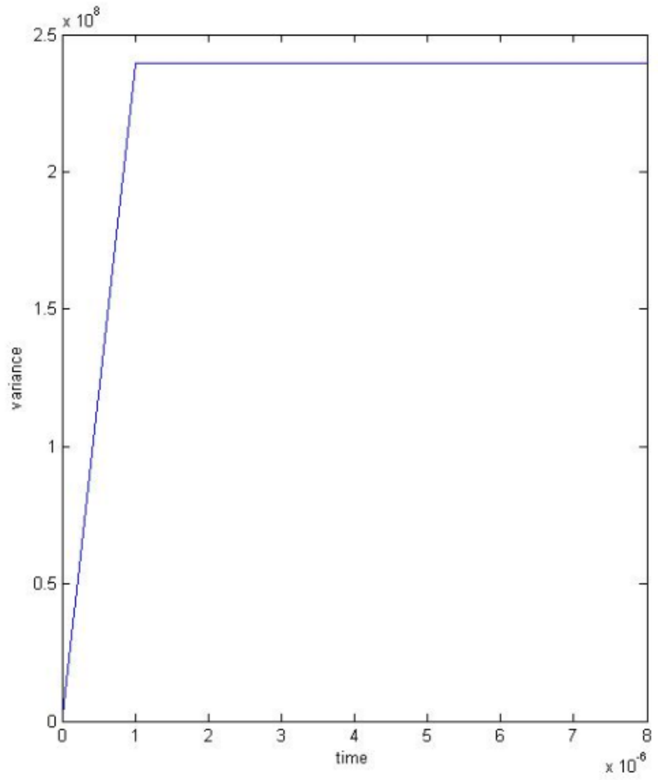

Fig.3.24. Variation of variance with time for CG amplifier

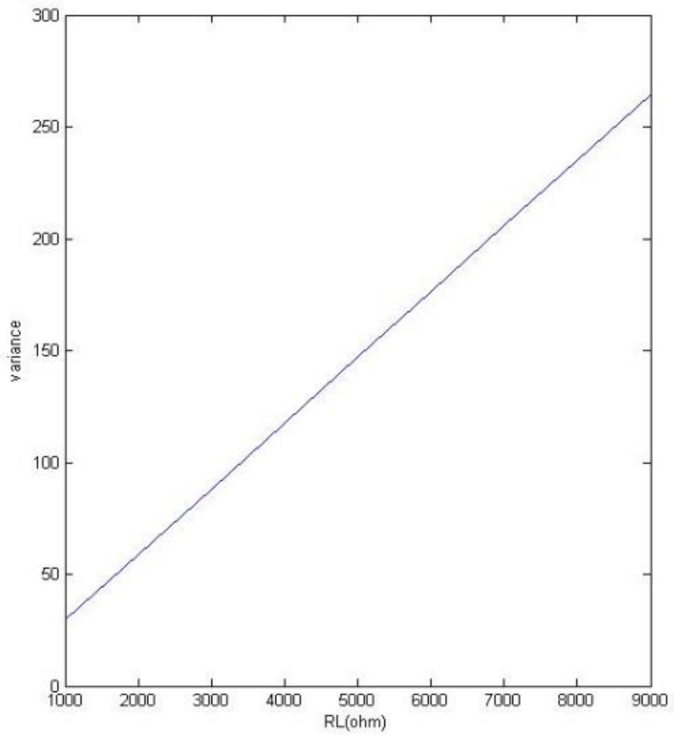

Fig.3.25. Variation of variance with R_L for CG amplifier

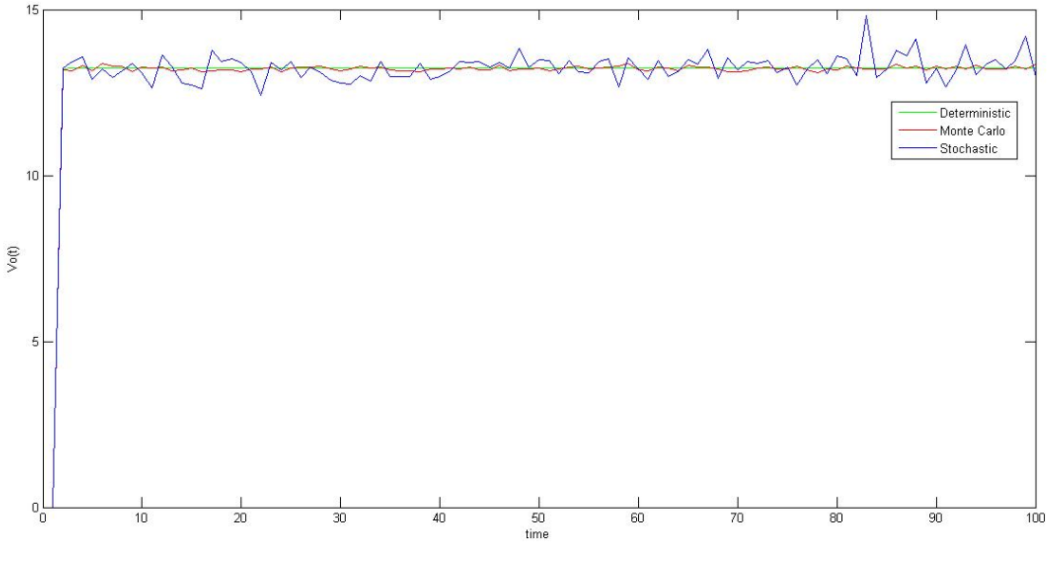

Fig.3.26. Comparison of deterministic, stochastic and Monte Carlo simulation for CG amplifier

3.6 STOCHASTIC DIFFERENTIAL EQUATION NOISE ANALYSIS OF COMMON-GATE AMPLIFIER WITH CAPACITIVE LOAD

The common-gate amplifier is an important circuit in analog design. There are varieties of applications of common- gate amplifier. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on the common- gate amplifier with capacitive load. The effect of noise is analysed at high frequencies.

The noise is assumed to be white Gaussian noise. Although it is an ideal condition, when noise is assumed as white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. A time domain method using SDE is used to analyse the effect of noise on the common- gate amplifier with capacitive load. The autocorrelation function of the output noise and other statistics like mean and variance are obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

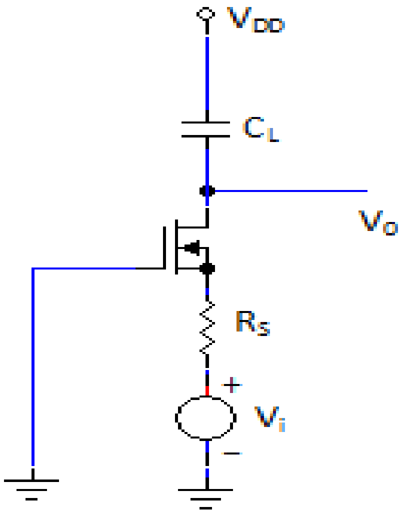

Fig.3.27. Common-Gate Amplifier with capacitive load

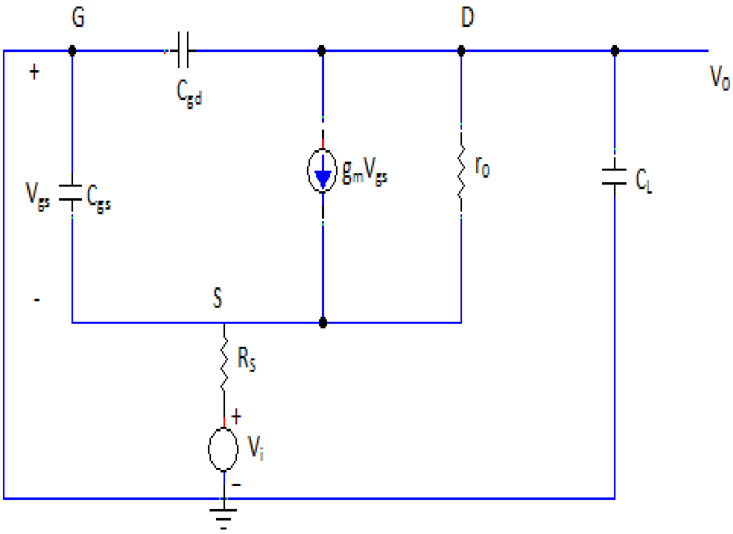

Fig.3.28. High-Frequency Equivalent Circuit of CG amplifier (capacitive load)

3.6.1 Noise Analysis using SDE

We consider common-gate amplifier with capacitive load which is shown Fig.3.27. Its high-frequency equivalent circuit is given in Fig.3.28. Now we will analyse this circuit using SDEs. If we assume that the value of R_s is very small, then $v_s = v_i$. From the circuit, we get

$$c_{gd}\frac{dv_0}{dt} + c_L\frac{dv_0}{dt} + \frac{v_0 - v_i}{r_0} = -g_m v_{gs} \tag{3.66}$$

Eq. (3.66) can be simplified further and written as

$$\frac{dv_0(t)}{dt} + kv_0(t) = gv_i \tag{3.67}$$

where $k = \frac{1}{(c_{gd} + c_L)}(\frac{1}{r_0})$ and $g = (g_m + \frac{1}{r_0})\frac{1}{(c_{gd} + c_L)}$. Consider $v_i(t) = \sigma n(t)$ with $n(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_i(t) = \sigma n(t)$ in eq. (3.67)

$$\frac{dv_0(t)}{dt} + kv_0(t) = g\sigma n(t)$$

$$dv_0(t) + kv_0(t)dt = g\sigma n(t)dt$$

Then we put $n(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_0(t) + kv_0(t)dt = g\sigma dW(t) \tag{3.68}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we take the expectation of both side of eq. (3.68)

$$dE[v_0(t)] + kE[v_0(t)]dt = E[g\sigma dW(t)] \tag{3.69}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (3.69) we obtain

$$\frac{dE[v_0(t)]}{dt} + kE[v_0(t)] = 0 \tag{3.70}$$

The solution of eq. (3.70) is written as

$$E[v_0(t)] = c_1 e^{-kt} \tag{3.71}$$

This is the mean of output process. Where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions.

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eq. (3.67) is rewritten

$$\frac{dv_0(t)}{dt} + kv_0(t) = gv_i \tag{3.72}$$

Now, consider eq. (3.72) $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_o(t)$ are zero at $t_1 = 0$. Both sides of eq. (3.72) is multiplied by $v_o(t_2)$ and then expectation is taken

$$\frac{dR_{v_0,v_0}(t_1,t_2)}{dt_1} + kR_{v_0,v_0}(t_1,t_2) = gR_{v_i,v_0}(t_1,t_2) \tag{3.73}$$

Again, consider eq. (3.72) at $t = t_2$ and we assume that the initial conditions for the correlation of $v_i(t)$ and $v_o(t)$ are zero at $t_2 = 0$. Both sides of eq. (3.72) is multiplied by $v_i(t_1)$ and then expectation is taken

$$\frac{dR_{v_i,v_0}(t_1,t_2)}{dt_2} + kR_{v_i,v_0}(t_1,t_2) = gR_{v_i,v_i}(t_1,t_2) \tag{3.74}$$

We know that $R_{v_i,v_i}(t_1,t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (3.74) is given as

$$R_{v_i,v_0}(t_1,t_2) = g\sigma^2 e^{k(t_1-t_2)} \tag{3.75}$$

Now, we put the value of $R_{v_i,v_o}(t_1,t_2)$ from (3.75) in eq. (3.73), we obtain

$$R_{v_0,v_0}(t_1,t_2) = \frac{g^2\sigma^2}{2k}\left(e^{-k(t_1-t_2)} - e^{-k(t_1+t_2)}\right) \tag{3.76}$$

When we substitute $t_1 = t_2 = t$ in eq. (3.76) we will have the second moment of the $v_o(t)$ that is variance of the output in this case

$$E[{v_0}^2(t)] = \frac{g^2\sigma^2}{2k}(1 - e^{-2kt}) \tag{3.77}$$

3.6.2 Simulation Results

To do the simulation of the above result, we used the following values of the parameters. $C_L = 2pf$, $r_o = 44k\Omega$, $\sigma = 0.25$, $c_{gd} = 2.8pF$, $g_m = 0.0016A/V$.

Fig.3.29 represents the variation of mean of output with time. It has observed from Fig.3.29 that the magnitude of mean of the output has peak value 0.01 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) and it reaches to steady state value of zero after 2μs (7μs in case of single ended input BJT differential amplifier [13]). The mean will be zero for zero initial conditions. Fig.3.30 represents the variation of variance of output with time. Variance increases linearly and reaches at constant value of 7.5×10^8 in 1μs. Fig.3.31 represents the Variation of variance with C_L. The variance decreases when C_L increases. It is observed that the time period of the signal will be less than the time during which the mean of the signal varies if the frequency of the input signal is more than 500 kHz (142.8 kHz for single ended input BJT differential amplifier [13]). So there will be more than 20 cycles (70 cycles for single ended input BJT differential amplifier [13]) of the signal with error in the results for the signals which have frequencies more than 10MHz.

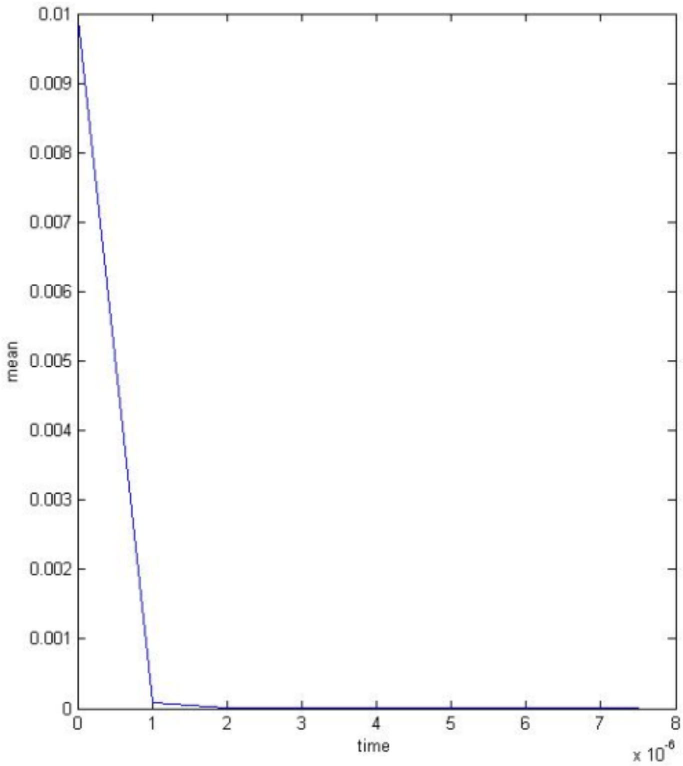

Fig.3.29. Variation of mean with time for CG amplifier (capacitive load)

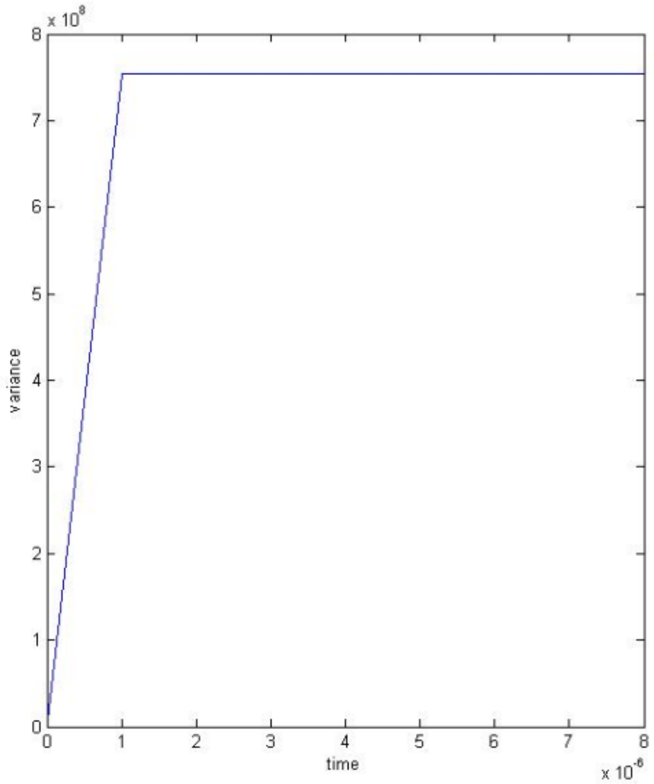

Fig.3.30. Variation of variance with time for CG amplifier (capacitive load)

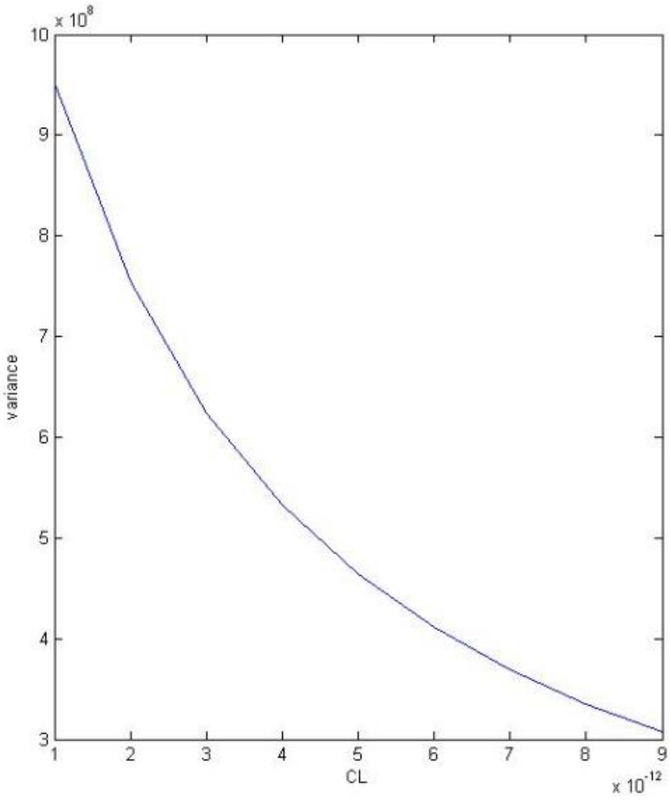

Fig.3.31. Variation of variance with C_L for CG amplifier (capacitive load)

3.7 Conclusion

Noise analysis of different FET amplifiers is done. Stochastic differential equation is used to do the external noise analysis for different amplifiers. Time domain method is used to obtain the solution of stochastic differential equations. Mean and variance of the output process is determined which may be useful in design process. It has observed that noise affects the considered circuits more at high input frequencies.

CHAPTER 4

NOISE ANALYSIS OF MOS DIFFERENTIAL AMPLIFIER USING SDE

4.1 INTRODUCTION

Noise analysis of MOS differential amplifiers is done in this chapter. The differential amplifier is an important circuit in analog design. There are varieties of applications of differential amplifier. The noises which can affect the performance of an amplifier are of two type, intrinsic noise and extrinsic noise. Intrinsic noise is generated within the amplifier and extrinsic noise may enter the circuit from the external disturbance. The effect of external noise is analysed on MOS differential amplifier.

In this thesis, a time domain method based on solving stochastic differential equation is used. To derive and compute non-Gaussian, non-stationary and nonlinear stochastic characterization of both amplitude and phase noise in an oscillator, the stochastic differential equation approach is adopted in [9]. The stochastic differential equation approach was adopted in [11] from simulation point of view for noise analysis. This method is based on linearization of stochastic differential equation about its simulated deterministic trajectory. In [12] noise analysis of sampling mixer is done. Three different sources of noise are analyzed. Conventional frequency domain method is used to analyze the external RF noise and intrinsic noise. Time domain method using stochastic differential equation is used to analyze the external local oscillator (LO) noise. In [13] noise analysis of single-ended input differential amplifier is performed using stochastic differential equation. Various statistics of output like mean and variance is obtained using stochastic differential equation. In [14] modeling of RC circuit is done to analyze the effect of external and internal noise. DC analysis of an RC circuit is done using first order ordinary differential equation and its stochastic analogues. In [15] noise analysis of simple single stage low-pass filter (SSLPF) with the fractional-order capacitor is performed with the help of stochastic differential equation. Various solution statistics of output like mean, variance is obtained using stochastic and fractional calculus. The change in statistics with the capacitor order is investigated. The closed form solutions of the step response of fractional filter are obtained.

The noise is assumed to be white Gaussian noise. Although it is an ideal condition, when noise is assumed as white Gaussian, it can be justified due to the presence of many random signal effects. As per central limit theorem, when there are additive effects of many random signals, the probability distribution of such random signals is Gaussian. It can be difficult to separate each term that produces randomness in the circuit, so the noise sources may be assumed to be white having flat power spectrum density.

Generally noise analysis is performed in frequency domain. When the circuit is linear and time invariant, this method is effective. But when noise analysis is done for extrinsic noise, the system can be either non-linear or time-variance because of switching behaviour of the signal. So frequency domain method is not applicable for extrinsic noise analysis. Time domain method using SDE is used to analyse the effect of noise on the MOS differential amplifier. The autocorrelation function of the output noise and other statistics like mean and variance is obtained using stochastic differential equations. An approach is used in which analytical solution of the SDE is obtained. The time varying nature of the circuit will be taken into account by analytical solution.

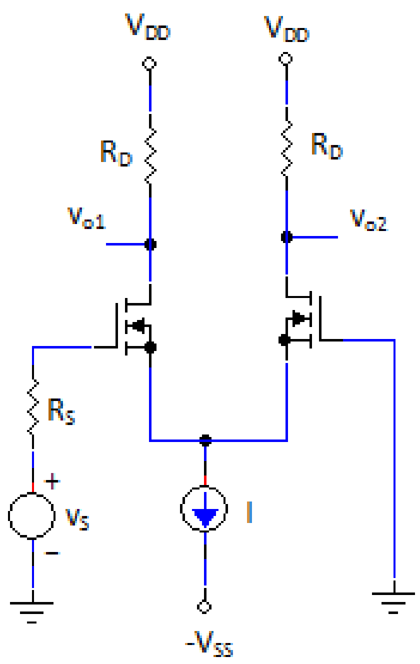

Fig. 4.1.MOS Differential Amplifier with one end grounded

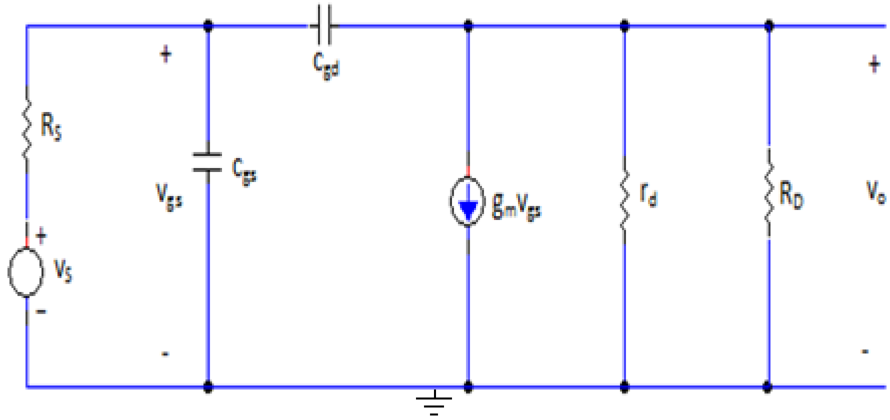

Fig.4.2.High-frequency equivalent half-circuit of MOS differential amplifier

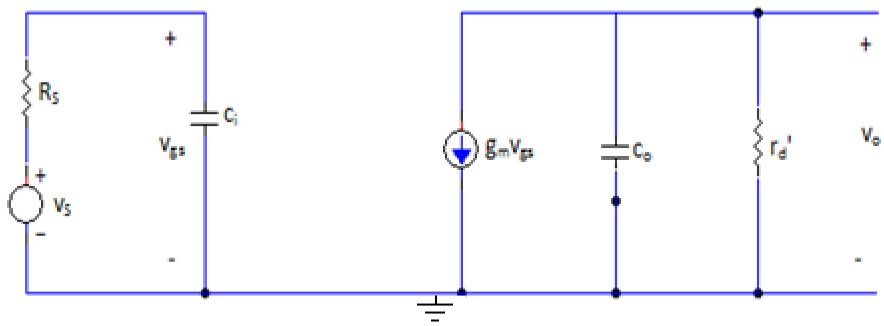

Fig.4.3.Simplified high-frequency equivalent half-circuit of MOS differential amplifier

4.2 NOISE ANALYSIS USING SDE

A differential amplifier is considered with one end grounded and the other end provided with the input signal which is shown in Fig.4.1. Fig.4.2 represents its high-frequency equivalent half-circuit. We assume that both MOS transistors are perfectly matched. We can obtain simplified equivalent circuit as shown in Fig.4.3, using Miller's Theorem and using some approximation. Now, we will analyse this circuit using SDE. From the simplified equivalent circuit, we obtain

$$\frac{v_s(t)-v_{gs}(t)}{R_s} = c_i\frac{dv_{gs}(t)}{dt} \qquad (4.1)$$

where $c_i = c_{gs} + c_{gd}(1 + g_m r_d{}')$, $c_o \approx c_{gd}$ and $r_d{}' = r_d R_D / r_d + R_D$.

$$c_o \frac{dv_o(t)}{dt} + \frac{v_o(t)}{r_d{}'} = -g_m v_{gs}(t) \tag{4.2}$$

Eq. (4.1) can be simplified further and written as

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_s(t)}{c_{iR_s}} \tag{4.3}$$

where $k_1 = 1/c_i R_s$. Consider $v_s(t) = \sigma\eta(t)$ with $\eta(t)$ as white Gaussian noise and σ^2 as the power spectrum density of noise at input. We put $v_s(t) = \sigma\eta(t)$ in eq. (4.3)

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{\sigma\eta(t)}{c_{iR_s}}$$

$$dv_{gs}(t) + k_1 v_{gs}(t)dt = \frac{\sigma\eta(t)dt}{c_{iR_s}}$$

Then we put $\eta(t)dt = dW(t)$ in the above equation, where $W(t)$ is considered as a Wiener process

$$dv_{gs}(t) + k_1 v_{gs}(t)dt = \frac{\sigma dW(t)}{c_{iR_s}} \tag{4.4}$$

(A) *Mean Analysis*: Mean is the first order statistic of any signal or process. It gives the average value of the signal. To obtain the mean of the output voltage, we first find the mean of $v_{gs}(t)$. We take the expectation of both side of eq. (4.4)

$$dE\left[v_{gs}(t)\right] + k_1 E\left[v_{gs}(t)\right]dt = \frac{E[\sigma dW(t)]}{c_{iR_s}} \tag{4.5}$$

For the Wiener process, $E[\sigma dW(t)] = 0$, so from eq. (4.5) we obtain

$$\frac{dE\left[v_{gs}(t)\right]}{dt} + k_1 E\left[v_{gs}(t)\right] = 0 \tag{4.6}$$

The solution of eq. (4.6) is written as

$$E\left[v_{gs}(t)\right] = c_1 e^{-k_1 t} \tag{4.7}$$

which is the mean of $v_{gs}(t)$, where c_1 is considered to be a constant, the value of which depends on the circuit's initial conditions. Now, eq.(4.2) is considered to obtain the mean of the output process. Simplify and take expectation of eq. (4.2), we obtain

$$\frac{dE[v_o(t)]}{dt} + \frac{E[v_o(t)]}{c_o r_d{}'} = \frac{-g_m E[v_{gs}(t)]}{c_o} \tag{4.8}$$

The solution of this is given by

$$E[v_o(t)]e^{k_2 t} = \frac{c_2}{k_2 - k_1}\left(e^{(k_2-k_1)t}\right) + c_3 \tag{4.9}$$

where $k_2 = 1/c_o r_d{}'$ and $c_2 = -g_m c_1/c_o$. c_3 is constant of integration which depends on circuit's initial conditions. Mean of output voltage will be zero if initial conditions are zero.

(B) *Variance Analysis*: Variance is the second order statistic of any signal or process. To obtain the variance of the output, we will determine the autocorrelation function for the output process. Initial conditions are considered to be zero for obtaining the autocorrelation of the output process. Eqs. (4.2) and (4.3) are rewritten

$$\frac{dv_o(t)}{dt} + k_2 v_o(t) = -\frac{g_m v_{gs}(t)}{c_o} \tag{4.10}$$

$$\frac{dv_{gs}(t)}{dt} + k_1 v_{gs}(t) = \frac{v_s(t)}{c_{iR_S}} \tag{4.11}$$

Eq. (4.10) is considered at time $t = t_2$ and we assume that the initial conditions for the autocorrelation of $v_o(t)$ are zero at $t_2 = 0$. Both sides of eq. (4.10) is multiplied by $v_o(t_1)$ and then expectation is taken

$$\frac{dR_{v_o,v_o}(t_1,t_2)}{dt_2} + k_2 R_{v_o,v_o}(t_1,t_2) = \frac{-g_m\, R_{v_o,v_{gs}}(t_1,t_2)}{c_0} \tag{4.12}$$

Again, eq. (4.10) is considered at time $t = t_1$ and we assume that the initial conditions for the correlation of $v_o(t)$ and $v_{gs}(t)$ are zero at $t_1 = 0$. Both sides of eq. (4.10) is multiplied by $v_{gs}(t_2)$ and then expectation is taken

$$\frac{dR_{v_o,v_{gs}}(t_1,t_2)}{dt_1} + k_2 R_{v_o,v_{gs}}(t_1,t_2) = \frac{-g_m\, R_{v_{gs},v_{gs}}(t_1,t_2)}{c_0} \tag{4.13}$$

Eq. (4.11) is considered at time $t = t_1$ and we assume that the initial conditions for the autocorrelation of $v_{gs}(t)$ are zero at $t_1 = 0$. Both sides of eq. (4.11) is multiplied by $v_{gs}(t_2)$ and then expectation is taken

$$\frac{dR_{v_{gs},v_{gs}}(t_1,t_2)}{dt_1} + k_1 R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{R_{v_s,v_{gs}}(t_1,t_2)}{c_i R_S} \tag{4.14}$$

Again, eq. (4.11) is considered at time $t = t_2$ and we assume that the initial conditions for the correlation of $v_s(t)$ and $v_{gs}(t)$ are zero at $t_2 = 0$. Both sides of eq. (4.11) is multiplied by $v_s(t_1)$ and then expectation is taken

$$\frac{dR_{v_s,v_{gs}}(t_1,t_2)}{dt_2} + k_1 R_{v_s,v_{gs}}(t_1,t_2) = \frac{R_{v_s,v_s}(t_1,t_2)}{c_i R_S} \quad (4.15)$$

Now, we want to have solutions of differential eqs. (4.12), (4.13), (4.14) and (4.15) to find out the value of $R_{v_o,v_o}(t_1,t_2)$. We know that $R_{v_s,v_s}(t_1,t_2) = \sigma^2\delta(t_1 - t_2)$, the solution of eq. (4.15) is given as

$$R_{v_s,v_{gs}}(t_1,t_2) = \frac{\sigma^2}{c_i R_S} e^{k_1(t_1-t_2)} \quad (4.16)$$

Now, we put the value of $R_{v_s,v_{gs}}(t_1,t_2)$ from eq. (4.16) in eq. (4.14), we obtain

$$R_{v_{gs},v_{gs}}(t_1,t_2) = \frac{\sigma^2}{2k_1(R_s c_i)^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \quad (4.17)$$

Now, we put the value of $R_{v_{gs},v_{gs}}(t_1,t_2)$ from eq. (4.17) in eq. (4.13), we obtain

$$R_{v_o,v_{gs}}(t_1,t_2) = \frac{k_3}{\frac{1}{r_d'} - k_1 c_o}\left(e^{\left(\frac{t_2-t_1}{c_o r_d'}\right)} - e^{\left(k_1 t_2 - \frac{t_1}{c_o r_d'}\right)} - e^{\left(-2k_1 t_2 + \frac{t_2-t_1}{c_o r_d'}\right)} + e^{\left(-k_1 t_2 - \frac{t_1}{c_o r_d'}\right)}\right) \quad (4.18)$$

where $k_3 = \frac{-g_m\sigma^2}{2k_1(R_s c_i)^2}$. Now, we put the value of $R_{v_o,v_{gs}}(t_1,t_2)$ from eq. (4.18) in eq. (4.12), we obtain

$$R_{v_o,v_o}(t_1,t_2) = \frac{-g_m k_3}{\frac{1}{r_d'} - k_1 c_o}\left(\left(e^{\frac{t_2-t_1}{c_o r_d'}} - e^{\frac{-t_1-t_2}{c_o r_d'}}\right)\frac{c_o r_d'}{2} + \frac{\left(e^{-2k_1 t_2 + \frac{t_2-t_1}{c_o r_d'}} - 2e^{k_1 t_2 - \frac{t_1}{c_o r_d'}} + e^{\frac{-t_1-t_2}{c_o r_d'}}\right)}{2\left(k_1 + \frac{1}{c_o r_d'}\right)} - \frac{\left(e^{-k_1 t_2 - \frac{t_1}{c_o r_d'}} - e^{\frac{-t_1-t_2}{c_o r_d'}}\right)}{\frac{1}{c_o r_d'} - k_1}\right) \quad (4.19)$$

When we substitute $t_1 = t_2 = t$ in eq. (4.19), we get the second order moment of $v_o(t)$, that is variance of the output in this case.

$$E[v_o^2(t)] =$$

$$\frac{-g_m k_3}{\frac{1}{r_d'} - k_1 c_o}\left(\left(1 - e^{\frac{-2t}{c_o r_d'}}\right)\frac{c_o r_d'}{2} + \frac{\left(e^{-2k_1 t} - 2e^{\left(k_1 - \frac{1}{c_o r_d'}\right)t} + e^{\frac{-2t}{c_o r_d'}}\right)}{2\left(k_1 + \frac{1}{c_o r_d'}\right)} - \frac{\left(e^{\left(-k_1 - \frac{1}{c_o r_d'}\right)t} - e^{\frac{-2t}{c_o r_d'}}\right)}{\frac{1}{c_o r_d'} - k_1}\right) \tag{4.20}$$

4.3 SIMULATION RESULTS

To do the simulation of the above result, we used the following values of the parameters. $R_D = 10k\Omega$, $R_S = 5k\Omega$, $r_d = 44k\Omega$, $g_m = 0.0016$, $\sigma = 0.25$, $c_{gs} = 3pF$, $c_{gd} = 2.8pF$.

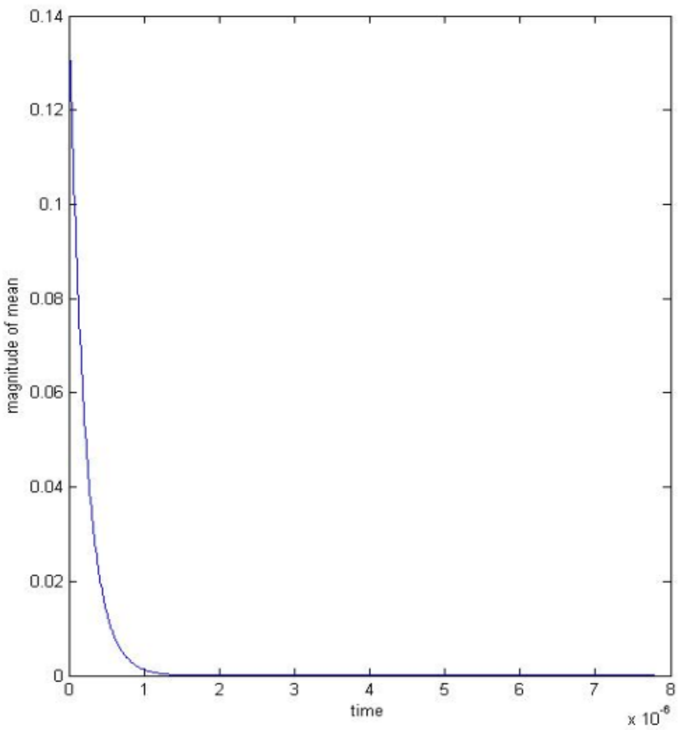

Fig.4.4. Variation of mean with time for MOS differential amplifier

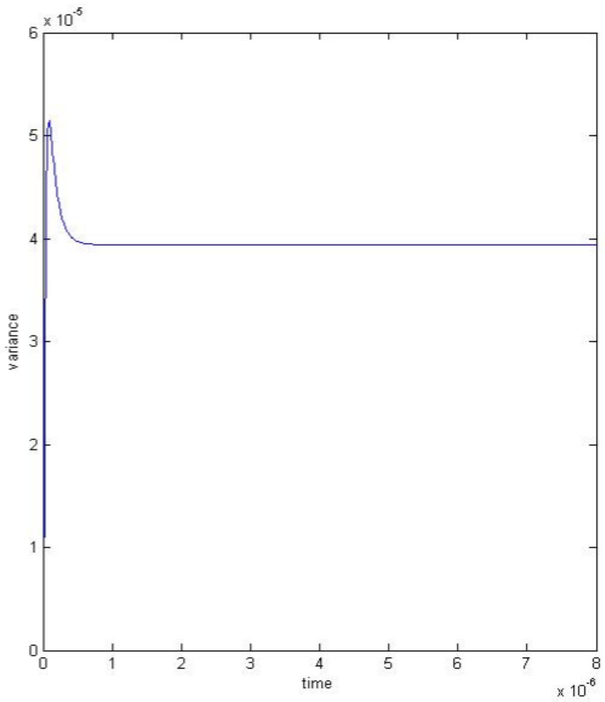

Fig.4.5 Variation of variance with time for MOS differential amplifier

Fig.4.4 represents variation of mean of the output with time for non-zero initial conditions ($v_o(0) = 0.01V$ and $v_{gs}(0) = 0.01V$). It can be observed from Fig.4.4 that the mean increases to peak value of 0.13 volts (12.375 volts in the case of single ended input BJT differential amplifier [13]) then it decreases to zero exponentially. It can also be observed that the mean becomes zero after around 1.5μs (7μs in the case of single ended input BJT differential amplifier [13]). Fig.4.5 represents the variation of variance with time which shows that it reaches the steady state value around 3.9×10^{-5} in 1μs. The variance is having maximum value around 5.1×10^{-5}. The maximum value of variance of the output is 1.8×10^{-3} in case of single ended input BJT differential amplifier [13].

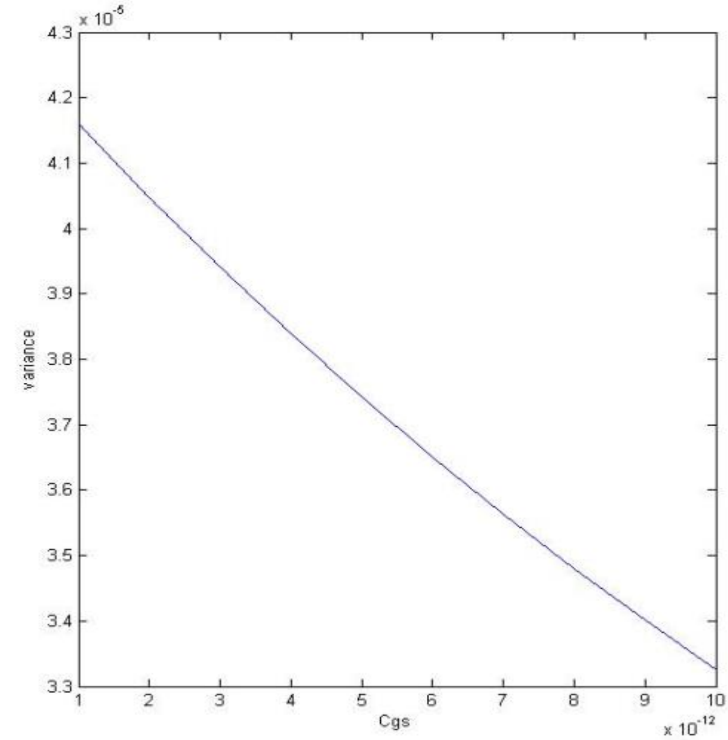

Fig.4.6. Effect of device parameter c_{gs} for MOS differential amplifier

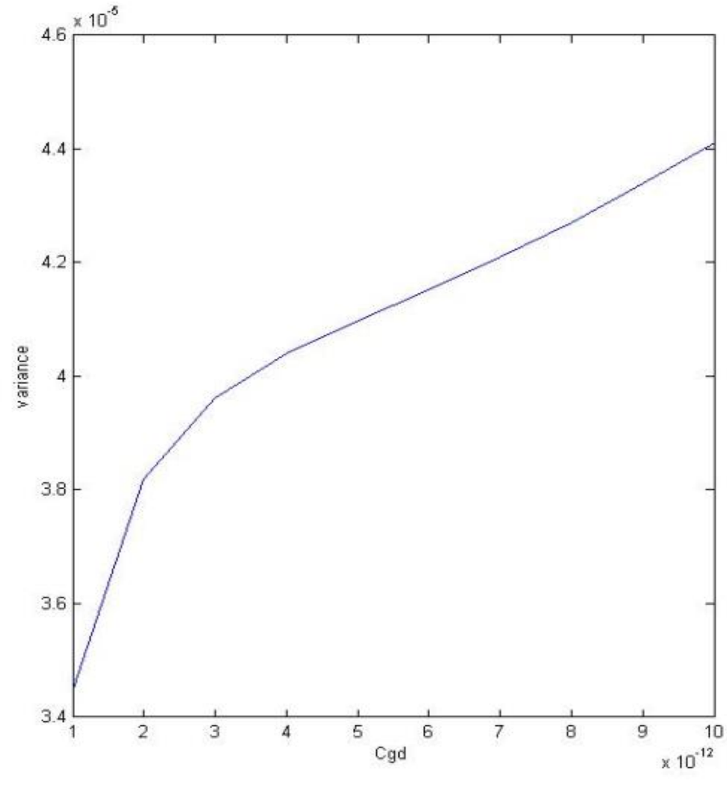

Fig.4.7. Effect of device parameter c_{gd} for MOS differential amplifier

From the analysis we can observe that if the frequency of the signal is more than 666.6 kHz (142.8 kHz for single ended input BJT differential amplifier [13]) then the time period of the signal will be less than the time during which the mean varies. So there will be more than 15 cycles (70 cycles for single ended input BJT differential amplifier [13]) of erroneous results for the signals which have frequencies more than 10MHz (as in the case of HDD application [30]).

It can also be observed that the variance may also be controlled by the MOS parameters. Fig.4.6 represents the variation of variance with c_{gs}. It shows that when the value of c_{gs} is increased the value of variance decreases. Fig.4.7 represents the variation of variance with c_{gd}. It shows that when the value of c_{gd} is decreased the variance decreases. We also analysed the circuit for variable load resistance. Fig.4.8 represents the variation of mean with load resistance. Fig.4.9 represents the variation of variance with load resistance.

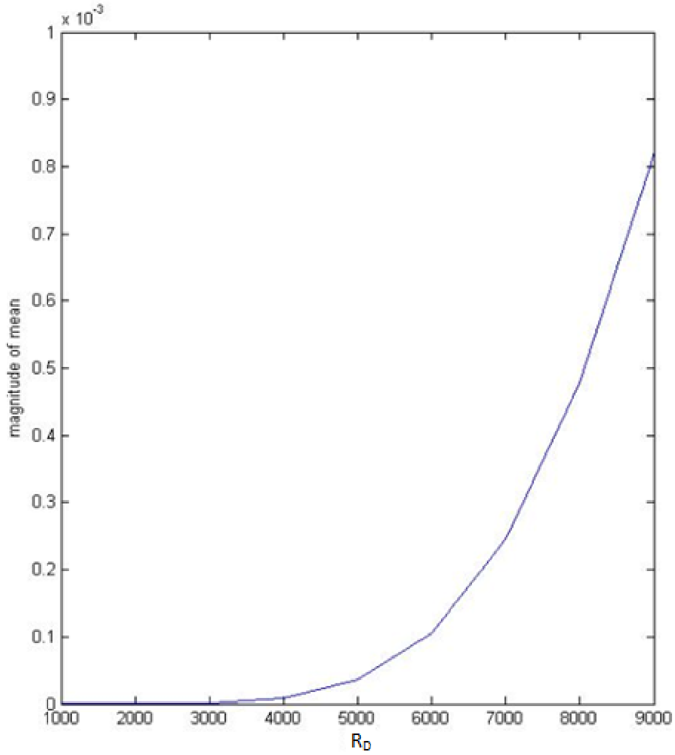

Fig.4.8. Variation of mean with load resistance for MOS differential amplifier

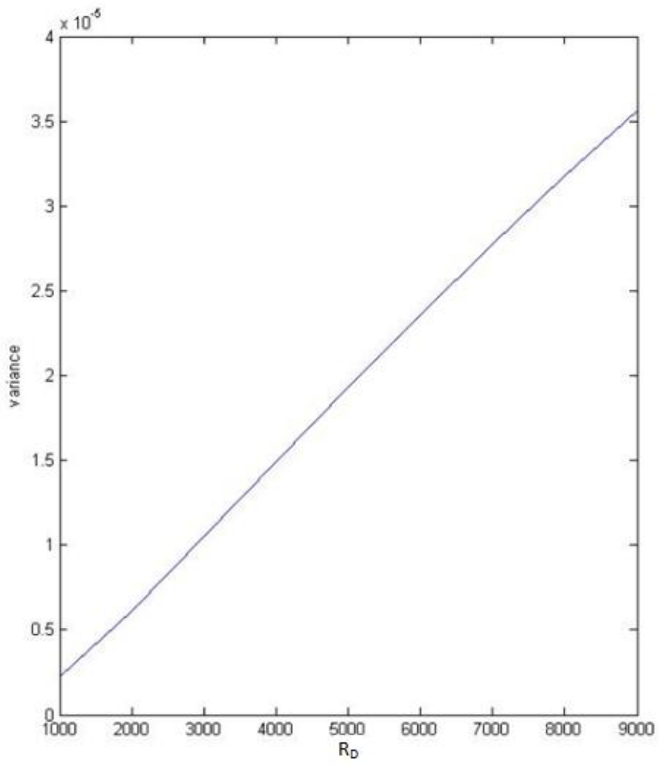

Fig.4.9 Variation of variance with load resistance for MOS differential amplifier

4.4 CONCLUSION

Noise analysis of MOS differential amplifier is done. Stochastic differential equation is used to do the external noise analysis for MOS differential amplifier. Time domain method is used to obtain the solution of stochastic differential equations. Mean and variance of the output process is determined which may be useful in design process. It has observed that noise affects the considered circuit more at high input frequencies. This circuit is also analysed for variable load resistance. Then the effects of device parameters on the noise performance of MOS differential amplifier are observed. Then comparison of the noise performance of BJT and MOS differential amplifier is done.

CHAPTER 5

CONCLUSION AND FUTURE DIRECTIONS

5.1 CONCLUSION

In this thesis, noise analysis of different BJT and FET amplifier circuits is performed using SDE. We also did the noise analysis of MOS differential amplifier. Time domain method is used to obtain the solution of stochastic differential equations. Mean and variance of the output process is determined which may be useful in design process.

In chapter 2, noise analysis of common base amplifier, common collector amplifier and common emitter amplifier BJT amplifiers is performed. These circuits are also analysed for variable load resistance. Mean and variance are determined for the circuits and observed that the noise affects the considered circuits more at high input frequencies. The results are also compared with the Monte Carlo simulation results. The results are very close to Monte Carlo simulation results. In chapter 3, noise analysis of common source amplifier, common drain amplifier and common gate amplifier FET amplifiers is performed. The analysis is also done for variable load resistance above mentioned amplifiers. Common source and common gate amplifier are analysed for capacitive load too. The results are also compared with the Monte Carlo simulation results. The results are very close to Monte Carlo simulation results. From the analysis it is concluded that the performance of the circuit degraded at high input frequencies. Analysis may be useful to have intermediate frequency amplifier, radio frequency amplifier and mixer in receiver circuits with better noise characteristics.

In chapter 4, the noise analysis of the MOS differential amplifier is performed. This analysis leads us to the conclusion that the noise affects the circuit more at high input frequencies. The analysis is also performed for variable load resistance for this circuit. Then we observed the effects of device parameters on the noise performance of the MOS differential amplifier. Noise performance of BJT and MOS differential amplifier is compared from which it is observed that as far as cycles of erroneous result is concern MOS differential amplifier has better performance than BJT differential amplifier.

5.2 FUTURE DIRECTIONS

The knowledge is ever expanding and so are the problems which the mankind strives to solve. In this spirit, it is hoped that the current activity will lead to further enhancements. The following are a few suggestions for further work in this area.

- In this thesis, we have focused our attention on stochastic differential equations driven by Brownian motion. More general kinds of noise can also be considered, for example, Poisson shot noise and more generally, a white non-Gaussian noise.
- Internal noise analysis can also be done by considering the randomness in the different components of the circuits. This can be done by adding the random term in the components of the circuits.
- Increasingly, for many application areas, it is becoming important to include elements of nonlinearity in order to model accurately the underlying dynamics of a system.

REFERENCES

[1] Vargas M, Pallas-Areny R. “On resistor-introduced thermal noise in linear circuits”, IEEE Transactions on Instrumentation and Measurement 2000; 49(1):87–88.

[2] Giannetti R. “On the thermal noise introduced by a resistor in a circuit”, IEEE Transactions on Instrumentation and Measurement 1996; 45(1):345.

[3] Itô K. “On stochastic differential equations” Memoirs of the American Mathematical Society 1951; 4:1–51.

[4] Gard TC. “Introduction to Stochastic Differential Equations”, Marcel Dekker: New York, 1988.

[5] Kalpazidou S. “Circuit duality for recurrent Markov processes”, Circuits, Systems, and Signal Processing 1995; 14(2):187–211.

[6] Kalpazidou S. “On the representation of finite Markov chains by weighted circuits”, Journal of Multivariate Analysis 1988; 25(2):241–271.

[7] Kalpazidou S. “Asymptotic behavior of sample weighted circuits representing recurrent Markov chains”, Journal of Applied Probability 1990; 27(3):545–556.

[8] Jazwinski AH. “Stochastic Process and Nonlinear Filtering Theory”, Academic Press: New York, 1970.

[9] Demir A., “Fully nonlinear oscillator noise analysis: an oscillator with no asymptotic phase”, International Journal of Circuit Theory and Applications 2007, 35:175–203.

[10] Finger L., “The Josephson junction circuit family: network theory”, International Journal of Circuit Theory and Applications 2000, 28(4):371–420.

[11] Demir A, Liu EWY, Vincentelli LS, “Time domain non-Monte Carlo noise simulation for nonlinear dynamic circuits with arbitrary excitations”, IEEE Transactions on Computer-Aided Design 1996, 15:493–505.

[12] Wei Yu and Bosco H.Leung, “Noise Analysis for Sampling Mixers Using Stochastic Differential Equation”, IEEE Transaction on circuit & system,VOL.46, NO.46, June 1999, 699-704.

[13] Tarun Kumar Rawat, Abhirup Lahiri, and Ashish Gupta, “Noise Analysis of Single-Ended Input Differential Amplifier using Stochastic Differential Equation”, World Academy of Science, Engineering and Technology 38 2008, 602-607.

[14] Tarun Kumar Rawat and Harish Parthasarathy, "Modelling of an RC Circuit Using a Stochastic Differential Equation", Thammasat Int. J. Sc. Tech.,Vol. 13. No. 2, April-June 2008, 40-47.

[15] Abhirup Lahiri and Tarun Kumar Rawat, "Noise analysis of single stage fractional-order low-pass filter using stochastic and fractional Calculus", ECTI Transaction on Electrical Eng., Electronics, and Communications, VOL.7, NO.2 August 2009, 136-143.

[16] A. Demir and A.L. Sangiovanni-Vincentelli, "Simulation and modeling of phase noise in open-loop oscillators", IEEE Custom Integrated Circuits Conf. 1996, 453-456.

[17] P. Bolcato and R. Poujois, "A new approach for noise simulation in transient analysis", in Proc. IEEE Int. Symp. Circuifs Syst., May 1992, pp. 887-890.

[18] J. A. McNeill, "Jitter in ring-oscillators", IEEE Journal of solid-State Circuits, Vol.32, No.6, June 1997, 870-879.

[19] A.G. Jordan and N.A. Jordan, "Theory of noise in metal oxide semiconductor devices", IEEE Trans. Electron Devices, Mar. 1965, 148-156.

[20] T.K. Rawat and H. Parthasarathy, "On stochastic modelling of linear circuits", Int. J. of Circuit Theory and Applications, 2008, Available Online, DOI: 10.1002/cta.560.

[21] T.E.Parker, " Characteristics and sources of phase noise in stable oscillators", 41st Annual frequency control symposium-1987, IEEE.

[22] B. Wang, J. R. Helums, and C. G. Sodini, "MOSFET thermal noise modeling for analog integrated circuits", IEEE **J.** Solid-State Circuits, vol. 29, 833-835, July 1994.

[23] P. J. Fish, "Electronic Noise and low noise Design", New York: McGraw-Hill, 1993.

[24] Kettani Houssaina, and BarmishB . Ross, "A New Monte Carlo Circuit Simulation Paradigm with Specific Results for Resistive Networks", in IEEE Transactions on Circuits and Systems I: Regular Papers' Vol.53, No.6, 1289-1299, 2006.

[25] Amit Mehrotra, "Noise Analysis of Phase-Locked Loops", IEEE transactions on circuits and systems—I: fundamental theory and applications, vol. 49, no. 9, september 2002, 1309-1316.

[26] Q. Su and K. Strunz, “Stochastic circuit modelling with Hermite polynomial chaos”, ELECTRONICS LETTERS 13th October 2005 Vol. 41 No. 21

[27] R. Rohrer, L. Nagel, R. Meyer, and L. Weber, “Computationally efficient electronic-circuit noise calculations,” IEEE **J.** Solid-state Circuits, vol. SC-6, No.4, Aug. 1971, 204-213.

[28] Jacob Millman and Christos C. Halkias, “Integrated Electronics”, Tata McGraw Hill, 1991.

[29] Adel S. Sedra, Kenneth C. Smith, adapted by Arun N. Chandorkar, “Microelectonic Circuits Theory and Applications”, Oxford University Press.

[30] Nodar, James (Dallas, Tx, US) Single ended preamplifier having improved noise characteristics, United States patent 20010053036 Texas Instruments Incorporated, http://www.freepatentsonline.com/20010053036.html.

[31] Andrew H. Zazwinski, “Stochastic Process and Nonlinear Filtering Theory”, Academic Press, New York, 1970

[32] W. A. Gardner, “Introduction to Random Processes with Applications to Signals & Systems”, 2nd ed. New York McGraw-Hill, 1990.

[33] G. R. Grimmet and D. R. Stirzaker, “Probability and Random Processes” New York, Oxford: Clarendon, 1992.

[34] Ross S. “Differential Equations”, (3rd edn). Wiley: New York, 1984.

[35] Sobczyk K. “Stochastic Differential Equations”, Kluwer Academic Publishers: Dordrecht, 1991.

[36] Kloeden PE, Platen E. “Numerical Solution of Stochastic Differential Equations”, Springer: Berlin, New York, 1992.

[37] I Karatzas and S.E. Shreve, “Brownian motion and Stochastic Calculus”, 2nd Edition, New York: Springer-Verlag, 1991.

[38] Z. Schuss, “Theory and Applications of Stochastic Differential Equations”, New York: Wiley, 1980.

[39] S.O. Rice “Mathematical Analysis of Random Noise”, Bell sys. tech. J. vols. 23 and 24.

[40] A. Papoulis Probability, “Random Variables and Stochastic Processes”, New York: McGraw Hill, 1965.

[41] K. S. Miller and B. Ross, “An Introduction to the Fractional Calculus and Fractional Differential Equations”, New York : John Wiley and Sons Inc.

[42] L. Arnold, “Stochastic Differential Equation”, John Wiley, NewYork.

[43] Hajimiri A, Lee TH. " A general theory of phase noise in electrical oscillators", IEEE Journal of Solid-State Circuits1998; 33(2):179 –194.

[44] Kaertner FX. " Analysis of white and $f^{-\alpha}$ noise in oscillators", International Journal of Circuit Theory and Applications 1990; 18:485 –519.

[45] Demir A, Mehrotra A, Roychowdhury J., " Phase noise in oscillators: a unifying theory and numerical methods for characterisation", IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications 2000; 47(5):655–674.

[46] Demir A., "Oscillator noise analysis", Proceedings of the 18th International Conference on Noise and Fluctuations, September 2005; 499–504.

[47] Demir A., " Phase noise in oscillators as differential-algebraic systems with colored noise sources", Proceedings of SPIE 2nd Annual Symposium on Fluctuations and Noise, Noise in Devices and Circuits II, vol. 5470, May 2004; 358–372.

[48] R. G. Meyer, L. Nagel, and *S.* K. Lui, "Computer simulation of *l / f* noise performance of electronic circuits," IEEE J. Solid-state Circuits,vol. *SSC-8,* pp. 237-240, June 1973.

[49] M. Okumura, H. Tanimoto, T. Itakura, and T. Sugawara, "Numerical noise analysis for nonlinear circuits with a periodic large signal excitation including cyclostationary noise sources," IEEE Trans. Circuits Systems I, vol. 40, pp. 581-590, Sept. 1993.

[50] Demir A. "Phase noise and timing jitter in oscillators with colored noise sources", IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications 2002; 49(12):1782–1791

[51] C. D. Hull and R. G. Meyer. "A systematic approach to the analysis of noise in mixers," IEEE Trans. Circuits Syst. I, vol. 40, pp. 909-919, Dec. 1993.

[52] S. Ruiz-Boque, J.Vall-Llosera, J.A. Novarro and G.Junjent, "Laser Noise impact in the design of an Heterodyne optical PSK receiver." Signal theory and communication department, E.T.S. Ingenieros de Telecommunication Barelona, Spain, 573-576.

[53] Mohammad M. Banat, " Statistical characterization of filtered phase noise in optical receiver." IEEE communication letter, Vol.7, No.2, February 2003.

[54] Katya Konstantinova Asparuhova, Elissaveta Dimitrova Gadjeva, " Noise analysis of operational amplifier circuit using MATLAB." 47th int'l spring seminar on Electronics Technology, IEEE, 2004, 471-475.

[55] N. Garmendia, J. Portilla, " Study of PM noise and noise figure in low noise amplifier working under small and large signal conditions", 1-4244-.0688-9/07/$20.00©2007 IEEE, 2095-2098.

[56] N. Ratier, L.Coutelean, R. Brendel, P.Guillemot, " AM and PM noise analysis in quartz crystal oscillator: Symbolic Calculus approach, 1998 IEEE international frequency control symposium, 156-163.

[57] G.Moradi, A.Ablipour, A Ghorbani, " An analytical method for noise analysis of a dual gate FET mixer." 2000 2nd international conference on microwave and millimetre wave technology proceedings, 92-95.

[58] Hiroshi Tanimoto and Ryuta Ito, " Noise analysis of Bipolar Harmonic Mixer." 47th IEEE International Midwest Symposium on Circuits and Systems, IEEE, 2004, 359-362.

[59] Stephen C.Terry, Benjamin J.Blalock, James M. Rochelle, M.Nance Ericson and Sam D.Caylor, " Time Domain Noise Analysis of Linear Time-Invariant and Linear Time-variant Systems using MATLAB and HSPICE." IEEE Transactions on Neclear Science, Vol. 52, No. 3, June 2005, 805-812.

[60] Dara C. OhAnnaidh and Thomas J.Brazil, " Time Domain Non-Linear Noise Analysis of FET oscillators." Department of Electronics & Electrical Engineering, University College, Dubin, Belfield, Dubin 4, Ireland,.

[61] M.M. Gourary, S.G. Rusakov, S.L. Ulyanov, M.M. Zharov, K.K. Gullakalli, B.J. Mulvaney, " A Numerical Technique for Time Domain Noise Analysis of Oscillators", 1-4244-1342-7/07/$25.00©2007 IEEE, 1002-1005.

[62] Suba Subramaniam, Alexander Petr, Michaelina Ong Ing Ing, " Unified Noise Characterization Technique for MOSFETs." ICNF 2017 978-1-5090-2760-6/17/$31.00©2017 IEEE.

[63] Edita Kolarova, " Modelling RL Electrical Circuits by Stochastic Differential Equation." Serbia & Montenogra, Belgrade, November 22-24, 2005, EUROCON 2005, 1236-1238.

[64] Lubomir Brancik, Edita Kolarova " Analysis of Higher-Order Electrical Circuits with Noisy Source via Stochastic Differential Equations approach." 22nd International Conference Radio Elektronika 2012.

[65] Edita Kolarova and Lubomir Brancik, " Vector Linear Stochastic Differential Equations and Their Applications to Electrical Networks." 978-1-4673-1118-2/12/$31.00©2012 IEEE, 311-315.

[66] Bronislovas Kaulakys, Julius Ruseckas, “Solution of nonlinear stochastic differential equation with 1/f noise power spectrum”, 2011 21st international conference on Noise and Fluctuations, IEEE, 192-195.
[67] Bronislovas Kaulakys, Rytis Kazakevicius, Julius Ruseckas, “ Modelling Gaussian and non-Gaussian 1/f noise by the linear stochastic differential equation” ICNF2013, 978-1-4799-0671-0/13/$31.00©2013 IEEE.
[68] Rawid Banchuin, Roungsan Chaisricharoen, “ The SDE based stochastic analysis of active filter”, WPMC 2019 - The 22nd international symposium on wireless personal multimedia communications, IEEE.
[69] Xiu-Ling WANG, Jia-Ying ZHANG, Wen-Lan WANG, Xiao-Dong ZHANG, “ Noise analysis for electronic circuit using Multisim”, 2010 2nd IEEE International Conference on Information Management and Engineering.
[70] Qinghang Zhao, Wenyu Sun, Yongpan Liu, Huazhong Yang, Jiaqing Zhao, Xiaojun Guo, “ Noise margin analysis for pseudo-CMOS circuits”, 978-1-5090-2687-6/16/$31.00©2016 IEEE.
[71] YIMING YU, KAI KANG, YIMING FAN, CHENXI ZHAO, HUIHUA LIU, YUNQIU WU, YONG-LING BAN AND WEN-YAN YIN, “ Analysis and design of inductorless wideband low-noise amplifier with noise cancellation technique”, 2169-3536©2017 IEEE, volume 5, 2017, 9389-9397.
[72] Xiao Wang, Zelin Shi, Yaohong Zhao, Jun Qiao, “ Noise analysis and optimization of a unity gain sampler”, 2017 2nd international Conference on Integrated Circuit and Microsystems, 978-1-5386-3506-3/17/$31.00©2017 IEEE, 78-81.
[73] Balaji G. Gawalwad, Shambhu N Sharma, “ Noise Analysis of a CMOS Inverter using the Ito Stochastic Differential Equation.” 2012 IEEE International Conference on Control Applications (CCA) Part of 2012 IEEE Conference on Systems and Control, October 3-5, 2012, Dubrovnik, Croatia, 344-349.

BRIEF BIO DATA OF THE RESEARCH SCHOLAR

DUSHYANT KUMAR SHUKLA is working as an Assistant Professor in the Electronics Engineering Department at J. C. Bose University of Science and Technology, YMCA, Faridabad, Haryana, India. He has passed his Bachelor of Engineering in Electronics & Telecommunication from Jabalpur Engineering College, Jabalpur in 2001, Master of Technology in Signal Processing from Netaji Subhash Institute of Technology, New Delhi in 2004. He is having around 17 years of teaching experience. He is currently doing research on signal processing. He has published over 10 research papers in various international journals.

LIST OF PUBLICATIONS

List of Published Paper in International Journal

S. No.	Title of the Paper	Name of the Journal	Volume & Issue	Year	Pages
1	Stochastic Differential Equation Noise Analysis of Common-Gate Amplifier with Capacitive Load	Journal of Semiconductor Devices and Circuits	Volume 5, Issue 1	2018	1-4
2	Noise Analysis of Single-Ended Input Differential Amplifier (MOS) using Stochastic Differential Equation	Journal of Microwave Engineering & Technologies	Volume 4, Issue 3	2017	19-25
3	Noise Analysis of Common-Collector Amplifier using Stochastic Differential Equation	International Journal of Engineering and Advanced Technology, Blue Eyes Intelligence Engineering & Science Publication Pvt. Ltd.	Volume-3, Issue-4	2014	92-94
4	Noise Analysis of Common-Emitter Amplifier using Stochastic Differential Equation	International Journal of Advanced Research in Computer and Communication Engineering, Tejass Publisheers	Vol. 3, Issue 3	2014	5649-5651
5	Noise Analysis of Common-Base Amplifier using Stochastic Differential Equation	International Journal of Electronics Communication and Computer Engineering, Timeline Publication Pvt. Ltd.	Volume 5, Issue 2	2014	381-383