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Roll No. .... Total Pages : 4

336302

December, 2019

B.Sc. (H) MATHEMATICS-III SEMESTER

Group Theory (BMH -302)

Time : 3 Hours Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) If G is a group, then identity element of G is unique. (1.5)

(b) Dihedral group D_3 is abelian group or not. Justify your answer. (1.5)

(c) If a and b are two elements of a group G, then $o(a) = o(xax^{-1}) = o(x^{-1}ax), \forall x \in G$. (1.5)

(d) Show that a finite group of order n containing an element of order n must be cyclic. (1.5)

(e) Let $H < G$ and $a, b \in G$. Prove that $Ha = H$ iff $a \in H$. (1.5)

(f) If H and K are normal subgroups of a group G and $H \cap K = (e)$, then $hk = kh$, for each $h \in H$ and $k \in K$. (1.5)

(g) If a cyclic subgroup H of G is a normal in G, then show that every subgroup of H is normal in G. (1.5)

(h) Find the external direct product of two cyclic groups :

$$G_1 = \{a, a^2 = e_1\}, \ G_2 = \{b, b^2, b^3 = e_2\}. \quad (1.5)$$

(i) Show that a finite cyclic group of order n is isomorphic to Z_n the group of integer modulo n. (1.5)

(j) Prove that a group G is abelian *iff* the mapping $f : G \to G$, given by $f(x) = x^{-1}$, is a homomorphism. (1.5)

PART - B

2. (a) If G is a group and if $a, b \in G$, show that $a.b = b.a \Rightarrow (a.b)^n = a^n.b^n$, n being any positive integer. (7)

(b) Let $G = \{(a, b) : a \neq 0, b \in R\}$ and $*$ be a binary composition defined by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G, *)$ is a non-abelian group. (8)

3. (a) If n is a positive integer, show that the set U_n of integers less than n and relatively prime to n is a group under multiplication mod n. (7)

(b) If $G = \langle a \rangle$ be a finite cyclic group of order n, then a^m is a generator of G *iff* $0 < m < n$ and $(m, n) = 1$. (8)

4. Show that the set A_n of all even permutation of S_n is a normal subgroup of S_n and $o(S_n) = \frac{n!}{2}$, where $n!$ denotes factorial n. (15)

5. (a) The order of a subgroup of a finite group divides the order of the group. (10)

(b) Prove that there is a one-to-one correspondence between any two right cosets of H in G. (5)

6. (a) Let H be a non-empty subset of a group G. Show that H is a normal subgroup of G *iff* $(gx)(gy)^{-1} \in H, \forall g \in G$ and $x, y \in H$. (8)

(b) If $f : G \to G'$ is a homomorphism, then $Ker f = \{e\} \Leftrightarrow f$ is one-to-one. (7)

7. If f is a homomorphism of G onto G' with kernel K, then $\frac{G}{Ker f} \approx G'$ or $\frac{G}{K} \approx G'$. (15)