Roll No. .... Total Pages : 4

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December, 2019

B.Sc. (H) (Mathematics) 1st SEMESTER

Calculus (BMH-101)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Find the derivatives of $y = 6 \sin h(x/3)$ w.r.t. appropriate variable.

   (b) Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.

   (c) Using L' Hospital rule, Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}$.

   (d) Using Reduction formula, evaluate $\int \sin^5 2x dx$.

(e) Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = 2/y$, $1 \leq y \leq 4$, about the y-axis.

(f) Find the focus and the directrix of the parabola $x^2 = 6y$.

(g) Using discriminant, classify the given equation $x^2 - 3xy + y^2 - x = 0$.

(h) Show that $u(t) = (\sin t)i + (\cos t)j + \sqrt{3}\, k$ has constant length and is orthogonal to its derivative.

(i) Find the unit tangent vector of the helix.

$r(t) = (6 \sin 2t)i + (6 \cos 2t)j + k$.

(j) If $r(t) = (\cos t)i + (\sin t)j + tk$, then find $\lim_{t \to \pi/4} r(t)$.

(1.5*10=15)

PART-B

2. (a) Use the steps of graphing procedure, graph the equation $y = x^2 - 4x + 3$. (7)

(b) Find all the asymptotes of the curve :

$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$. (8)

3. (a) The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis, and the line $y = 1$ is revolved about the line $x = 3/2$ to generate a solid. Find the volume of the solid. (7)

(b) Find the arc length of the curve $y = x^{3/2}$ from the point (1, 1) to $(2, 2, \sqrt{2})$. (8)

4. (a) The x-axis and y-axis are rotated through an angle $\pi/4$ radians about the origin. Find the equation for the hyperbola $2xy = 9$ in the new coordinates. (7)

(b) Find the polar equation of the circle

$(x - 3)^2 + (y + 1)^2 = 4$. (8)

5. (a) Show that if $r(t) = f(t)i + g(t)j + h(t)k$ is differentiable at $t = t_0$, then it is continuous at t_0 as well. (7)

(b) Prove that if u and v are differentiable functions of t, then

(i) $\frac{d}{dt}(u+v) = \frac{du}{dt} + \frac{dv}{dt}$,

(ii) $\frac{d}{dt}(u-v) = \frac{du}{dt} - \frac{dv}{dt}$. (8)

6. (a) Using Leibnitz's rule find the nth derivative of $y = e^x x^2$. (7)

(b) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq 1/2$, about the x-axis. (8)

7. (a) Show that the point $(2, \pi/2)$ lies on the curve $r = 2 \cos 2\theta$. (7)

(b) Show that the radius of curvature at the end of the major axis of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the semi-latus rectum of the ellipse. (8)
