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# 336102

**December, 2019**

**B.Sc. (H) Mathematics- I SEMESTER**

**Algebra (BMH-102)**

Time : 3 Hours] [Max. Marks : 75

*Instructions :*

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART - A

**1.** (a) Find the polar representation of the following complex number

$$-\frac{1}{2}-i\frac{\sqrt{3}}{2}. \qquad (1.5)$$

(b) Write down all the values of $(1 + i)^{1/3}$. (1.5)

(c) State De-Moivre's theorem for integral indices. (1.5)

(d) Show that $R$ is an equivalence relation on $Z$ defined as $aRb$ if and only if $a - b$ is divisible by 5. (1.5)

(e) State Division algorithm for integers. (1.5)

(f) Prove that $8^n - 3^n$ is a multiple of 5 for all integers $n \geq 1$. (1.5)

(g) Write the matrix equation $AX = b$ as a vector equation

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix}$$ (1.5)

(h) Determine if the following system is consistent :

$$x + 4y = 5$$

$$x + 3y + 5z = -2$$

$$3x + 7y + 7z = 6.$$ (1.5)

(i) Prove that $T(0) = 0$ where $T : R^n \to R^m$ is a linear transformation. (1.5)

(j) Evaluate $T\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ where $T : R^2 \to R^3$ is a linear transformation such that $T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. (1.5)



## PART - B

**2.** (a) Find $|z|$, arg $z$, arg $\bar{z}$, arg $(-z)$ if $z = (6 - 6i)(1 + i)$. (8)

(b) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, prove that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$.

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$. (7)

**3.** (a) Solve $z^6 + iz^3 + i - 1 = 0$. (8)

(b) Show that $f$ has an inverse where $f : Z \to N$ be defined as $f(x) = \begin{cases} 2|x| & \text{if } x < 0 \\ 2x+1 & \text{if } x \geq 0 \end{cases}$. Also evaluate $f^{-1}(5806)$. (7)

**4.** (a) Prove that for any set $A$, $|A| < |P(A)|$, where $P(A)$ is the power set of $A$. (8)

(b) Prove that the linear congruence relation $ax \equiv b \pmod{n}$ has solution if and only if gcd $(a, n)$ divides $b$. (7)

5. (a) Reduce the following matrix to row echelon form and then to reduced echelon form

$$\begin{bmatrix} 2 & -3 & 5 & 1 \\ 3 & 1 & -1 & 2 \\ 1 & 4 & -6 & 1 \end{bmatrix}$$

(8)

(b) Find the values of λ and μ such that the following system of equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have

(i) No solution

(ii) Unique solution

(iii) Infinitely many solutions (7)

6. (a) Check whether the following vectors are linearly dependent? If they are linearly dependent, find a relation between them

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 9 \\ 3 \end{bmatrix}.$$

(8)



(b) Find $T\begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}$, if the standard matrix of a linear transformation $T$ is $\begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & 2 & 4 \\ -1 & -3 & 2 & 0 \end{bmatrix}$. (7)

7. (a) Find the characteristic polynomial, eigen values and corresponding eigen vectors of the following matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & 3 \end{bmatrix}$. (8)

(b) Find the rank and nullity of the following matrix

$$\begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

(7)