5. (a) Obtain an expression for the divergence of vector in orthogonal curvilinear coordinates and express it in cylindrical co-ordinates. (7)

(b) Prove that Spherical polar co-ordinate system is orthogonal. (8)

6. Solve by method of variation of parameters :

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}. \quad (7)$$

(b) Solve the differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x\log x. \quad (8)$$

7. (a) Find the constants a,b,c so that :

$$\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$$

is irrotational and hence find function ϕ such that :

$$\vec{F}\nabla\phi. \quad (7)$$

(b) Show that the vector field

$$\vec{F}=2x(y^2+z^3)\hat{i}+2x^2y\hat{j}+3x^2z^2\hat{k}$$

is conservative. Evaluate the work done in moving a particle from (–1, 2, 1) to (2, 3, 4). (8)

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May 2026

B.Sc. (Physics) II SEMESTER

Mathematical Physics–I (PHU–102–V)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any **four** questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find the area of a parallelogram whose adjacent sides are :

$$\hat{i}-2\hat{j}+3\hat{k} \text{ and } 2\hat{i}+\hat{j}-4\hat{k}. \quad (1.5)$$

(b) If $\phi=3x^2y-y^3z^2$; find grad ϕ at the point $(1,-2,-1)$. (1.5)

(c) If a force $\vec{F}=2x^2y\hat{i}+3xy\hat{j}$ displaces a particle in the xy-plane from (0, 0) to (1, 4) along a curve $y = 4x^2$. Find the work done. (1.5)

(d) State Divergence Theorem. (1.5)

(e) Solve:

$$(D^2 - 6D + 25)^2 y = 0.$$ (1.5)

(f) Find the angle between the surface :

$$x^2 + y^2 + z^2 = 9 \text{ and } x^2 + y^2 - z = 3$$

at (2, –1, 2). (1.5)

(g) Find 'a' so that the vectors :

$$\left(2\hat{i} - \hat{j} + \hat{k}\right); \left(\hat{i} + 2\hat{j} - 3\hat{k}\right); \left(3\hat{i} + a\hat{j} + 5\hat{k}\right)$$

are coplanar. (1.5)

(h) Evaluate :

$$\int_{-\infty}^{\infty} \sin 2t \; \delta\left(t - \frac{\pi}{4}\right) dt.$$ (1.5)

(i) Define Dirac-Delta Function and write any of its two properties. (1.5)

(j) Assuming :

$$\vec{a}.\left(\vec{b} \times \vec{c}\right) \neq 0 \text{ and } \vec{d} = x\vec{a} + y\vec{b} + z\vec{c}.$$

Find the value of x, y and z. (1.5)

PART–B

2. (a) State and verify Green's Theorem in the plane for curve C :

$$\oint \left(3x^2 - 8y^2\right) dx + (4y - 6xy) dy,$$

where C is the boundary of the region defined by :

$$y = \sqrt{x} \text{ and } y = x^2.$$ (7)

(b) Find the directional derivative of :

$$\phi = 4e^{2x - y + z} \text{ at } (1, 1, -1)$$

in the direction towards the point (–3, 5, 6). (8)

3. (a) Verify Stoke's Theorem for :

$$\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - (xz)\hat{k},$$

over the surface of a cube :

$$x = 0, \; y = 0, \; z = 0, \; x = 2, \; y = 2, \; z = 2$$

above the XOY plane with bottom open. (7)

(b) Show that :

$$\text{div (grad } r^n) = n(n+1)r^{n-2}, \text{ where } r = \sqrt{x^2 + y^2 + z^2}.$$

(8)

4. (a) Solve :

$$\left(D^2 + 5D + 6\right) y = 2x \; e^{2x} + 3e^x \cos 2x.$$ (7)

(b) Solve using the method of undetermined coefficients the equation:

$$\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 25y = 14e^{-5x}.$$ (8)