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May 2026

B.Sc. (Physics) IV SEMESTER

Mathematical Physics–II (PHU-202-V)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Define periodic function with an example. (1.5)

(b) Prove that :

$$2J_n' = J_{n-1} - J_{n+1}.$$ (1.5)

(c) Prove $J_n(kx)$ is the solution of :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (k^2x^2 - n^2)y = 0$$

differential equation. (1.5)

(d) Determine the ordinary, regular and irregular singular points for the differential equation :

$$2x^2(1-x)^2y'' + (1+x)y' - x^2y = 0.$$ (1.5)

(e) Prove that :

$$\Gamma(n+1) = n\Gamma(n).$$ (1.5)

(f) State Parseval's Identity. (1.5)

(g) State Orthogonal property of Hermite polynomials. (1.5)

(h) Write Dirichlet Conditions for Fourier series. (1.5)

(i) Find the value of :

$$\beta\left(\frac{3}{2},\frac{1}{2}\right).$$ (1.5)

(j) Find the value of :

$$J_{-1}(x) + J_1(x).$$ (1.5)

PART–B

2. (a) Determine the Fourier series of function :

$$f(x) = x\cos x, -\pi < x < \pi.$$ (8)

(b) Deduce Euler's formulae for the determination of Fourier coefficients. (7)

3. (a) Obtain a cosine series for

$$f(x) = 1 + x \text{ in } 0 \le x \le 2,$$

and hence deduce that :

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$$ (7)

(b) Solve the following differential equation, about $x = 0$, using Power Series Solution method :

$$(1-x^2)y'' - xy' + 4y = 0.$$ (8)

4. (a) Solve the following differential equation using Frobenius method :

$$x^2y'' + (x^2 + x)y' + (x-9)y = 0.$$ (8)

(b) Solve the differential equation $3xy'' + 2y' + y = 0$ using Frobenius method. (7)

5. (a) State and proof orthogonality of Bessel Function. (8)

(b) Show that :

$$\sqrt{\left(\frac{1}{2}\pi x\right)}J_{3/2}(x) = \frac{\sin(x)}{x} - \cos(x).$$ (7)

6. (a) Evaluate :

$$\int_0^\infty \frac{x^a}{a^x}dx, \text{ hence show that } \int_0^\infty \frac{x^7}{7^x}dx = \frac{7!}{(\log 7)^8}.$$ (8)

(b) Determine generating function of Hermite polynomial using Rodrigue Formula. (7)

7. (a) Prove that :

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{\beta(m,n)}{a^n(1+a)^m}.$$ (5)

(b) Prove that :

$$H_{2n+1}(0) = 0.$$ (5)

(c) Evaluate :

$$\int_0^1 x^{n-1}\left[\log_e\left(\frac{1}{x}\right)\right]^{m-1}dx.$$ (5)