Sr. No. 323605

May 2026

B.Sc. (Mathematics/ Mathematics & Comp.) – 6th Sem BMH24-606

Tensor Tools for Mathematical Modelling (VAC24-M-601/MTU-316-V)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define summation convention with the help of an example. (1.5)

(b) What are dummy suffixes? (1.5)

(c) Find the value of δ_{ii}. (1.5)

(d) Define cartesian tensor of second order. (1.5)

(e) Define isotropic tensor. (1.5)

(f) Define second invariant of tensor A. (1.5)

(g) Write the mathematical expression for the gradient of a scalar field in comma notation. (1.5)

(h) Define strain tensor. (1.5)

(i) Define deformation gradient tensor. (1.5)

(j) Prove that $(\vec{u}\cdot\nabla)\vec{v} = (\nabla\vec{v})\vec{u}$ (1.5)

PART -B

Q2 (a) Write $a_{ik}x_k x_j = b_{ij}$ in expanded form. (7)

(b) Using the usual properties of δ –symbol, prove the vector identity (8)

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})$$

Q3 (a) Discuss briefly the Levi-Civita ε –symbol. How is it related to the scalar triple product of the usual unit vectors? (7)

(b) Write down the following equations in a condensed form: (8)

$$a_{11} = \alpha\,(b_{11}+b_{22}+b_{33}) + \beta\, b_{11}$$
$$a_{22} = \alpha\,(b_{11}+b_{22}+b_{33}) + \beta\, b_{22}$$
$$a_{33} = \alpha\,(b_{11}+b_{22}+b_{33}) + \beta\, b_{33}$$
$$a_{12} = \beta\, b_{12},\quad a_{23} = \beta\, b_{23},\quad a_{31} = \beta\, b_{31}$$
$$a_{21} = \beta\, b_{21},\quad a_{32} = \beta\, b_{32},\quad a_{13} = \beta\, b_{13}$$

Q4 The x_i' system is obtained by rotating the x_i system about the x_3 axis through an angle θ in the sense of the righthanded screw. Find the transformation matrix. If a point P has coordinates (1,1,1) in the x_i system, find its coordinates in the x_i' system. If a point Q has coordinates (1,1,1) in the x_i' system, find its coordinates in the x_i system. (15)

Q5 (a) Write down $\text{div}(\vec{u} \times \vec{v})$ in suffix notation and hence prove the identity (7)

$$\text{div}(\vec{u} \times \vec{v}) = \vec{v} \cdot \text{curl}\, \vec{u} - \vec{u} \cdot \text{curl}\, \vec{v}.$$

(b) Let $\vec{u}$ be a given vector, and $\boldsymbol{A}$ be a given tensor. For any fixed vector $\vec{c}$ prove the following: (8)

i) $(\nabla \vec{u}^T)\vec{c} = \nabla(\vec{u} \cdot \vec{c})$
ii) $(\text{div}\, \boldsymbol{A}) \cdot \vec{c} = \text{div}(\boldsymbol{A}^T \vec{c})$
iii) $(\text{curl}\, \boldsymbol{A})\vec{c} = \text{curl}(\boldsymbol{A}^T \vec{c})$

Q6 (a) Write the brief note on the generalized Hooke's Law. (7)

(b) Prove the following: (8)

i) $x_{i,j} = \delta_{ij}$
ii) $x_{i,i} = 3$
iii) $(x_m x_n)_{,i} = \delta_{im} x_n + \delta_{in} x_m$
iv) $(x_m x_n)_{,ij} = \delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}$

Q7 Define stress tensor. The stress matrix at a certain point P in a material is given by (15)

$$[\tau_{ij}] = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & -5 \\ 4 & -5 & 0 \end{bmatrix}$$

Find

i) the stress vector on a plane element p through P and parallel to the plane $2x_1 + x_2 - x_3 = 1$
ii) the magnitude of stress vector
iii) the angle that the stress vector makes with the normal to the plane.