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December, 2019

B.Sc. (H) Mathematics - III SEMESTER

Theory of Real Functions (BMH-301)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Show that $\lim_{x \to c} x^2 = c^2$ for any $c \in R$. (1.5)

(b) Evaluate the following limit or show that it does not exist $\lim_{x \to 1+} \frac{x}{x-1} (x \neq 1)$. (1.5)

(c) State Discontinuity criteria for a function. (1.5)

(d) Prove that if $f : I \to R$ has a derivative at $c \in I$, then f is continuous at c. (1.5)

(e) State Non-uniform continuity criteria for a function. (1.5)

(f) Define absolute maximum point and absolute minimum point for a function. (1.5)

(g) State Rolle's Theorem. (1.5)

(h) For the following function find the interval on which the function is increasing and those on which it is decreasing : $f(x) = x^2 - 3x + 5$. (1.5)

(i) Define convex function. (1.5)

(j) Approximate $\sqrt[3]{1+x}$, $x > -1$ with $n = 2$ by Taylor's theorem. (1.5)

PART - B

2. (a) Prove that if $f : A \to R$ and c is a cluster point of A, then f can have only one limit at c. (8)

(b) Prove that if $A \subseteq R$, f and g are functions on A to R, $b \in R$ and $c \in A$ and that f and g are continuous at c, then $f + g$, $f - g$, fg and bf are continuous at c. (7)

3. (a) Prove that if $A \subseteq R$, f and g are functions on A to R, $c \in R$ be a cluster point of A, $f(x) \le g(x)$ for all $x \in A$, $x \ne c$ and $\lim\limits_{x \to c} f = \infty$ then $\lim\limits_{x \to c} g = \infty$. (8)

(b) State and prove Caratheodory's Theorem. (7)

4. (a) Prove that if $f : I \to R$ be continuous on I, where I is a closed bounded interval then f is uniformly continuous on I. (8)

(b) Prove that if $f : I \to R$ be continuous on I, where $I = [a, b]$ is a closed bounded interval then f has an absolute maximum and absolute minimum on I. (7)

5. (a) State and prove Mean value theorem. (8)

(b) Prove that if f is differentiable on $I = [a, b]$ and if k is a number between $f'(a)$ and $f'(b)$ then there is at least one point $c \in (a, b)$ such that $f'(c) = k$. (7)

6. (a) Prove that if $f : I \to R$ be differentiable on the interval I, then f is increasing on I if and only if $f'(x) \ge 0$ for all $x \in I$. (8)

(b) State and prove Taylor's Theorem. (7)

7. (a) Prove that if f and g are continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) \neq 0$ for all $x \in (a, b)$ then there exist $c \in (a, b)$ such that

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}.$$ (8)

(b) Prove that if $f : I \to R$ have a second derivative on I, where I is an open interval, then f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. (7)