Roll No. ....

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336201

May 2019

B.Sc. (Mathematics) 2nd SEMESTER

REAL ANALYSIS

(BMH-201)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Define Open set and closed set. (1.5)

(b) Determine the set A of all real numbers x such that $2x + 3 \leq 6$. (1.5)

(c) State Bernoulli's Inequality. (1.5)

(d) If $y > 0$, then show that there exists $n_y \in \mathbb{N}$ such that $n_y - 1 \leq y \leq n_y$. (1.5)

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(e) Define Limit point of a set. (1.5)

(f) Show that $\lim\left(\frac{3n+2}{n+1}\right)=3$. (1.5)

(g) Show that a convergent sequence of real numbers is bounded. (1.5)

(h) Show that if X and Y are sequences such that X and X + Y are convergent, then Y is convergent. (1.5)

(i) Show that the series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+\cdots\cdots$ ... ... is convergent. (1.5)

(j) Test for convergence of the series $\sum\frac{n^2-1}{n^2+1}x^n$, $x>0$. (1.5)

PART–B

2. (a) State and Prove Cantor's Theorem. (8)

(b) If a, b, c be any elements of R then

(i) If $a > b$ and $b > c$, then prove that $a > c$.

(ii) If $a > b$, then prove that $a + c > b + c$. (7)

3. (a) State and Prove Completeness Property of R. (8)

(b) State and Prove Bolzano-Weierstrass Theorem for sets. (7)

4. (a) State and Prove Archimedean Property. (8)

(b) Prove that there exists a positive real number x such that $x^2 = 2$. (7)

5. (a) State and prove Cauchy Convergence Criterion for convergence of a sequence. (8)

(b) Let $X = \{x_n\}$ be the sequence of real numbers that converges to x and suppose $x_n \geq 0$. Then prove that the sequence $\{\sqrt{x_n}\}$ of positive square roots converges and $\lim\{\sqrt{x_n}\}=\sqrt{x}$. (7)

6. (a) If $X=\{x_n\}$ converges to x and $Z=\{z_n\}$ is a sequence of non-zero real numbers that converges to z and if $z \neq 0$, then show that the quotient sequence $\frac{x}{z}$ converges to $\frac{x}{z}$. (8)

(b) Test for the convergence of series

$$\frac{\alpha}{\beta}+\frac{(1+\alpha)}{(1+\beta)}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\cdots\cdots\cdots \quad (7)$$

7. (a Prove that every Absolutely convergent series is convergent. (8)

(b) Apply Cauchy's Integral Test to test the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$. (7)
