5. (a) Find the orthogonal trajectories of the family of parabolas $y = cx^2$. (7)

(b) Write a note on the exponential growth and exponential decay model. (8)

6. (a) Find the general and singular solutions of

$$9p^2(2-y)^2 = 4(3-y), \text{ where } p = \frac{dy}{dx}. \quad (7)$$

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x. \quad (8)$$

7. (a) Solve the differential equation :

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^x. \quad (7)$$

(b) (i) Let k be the rate constant for a radioactive element and τ be the half-life of the radioactive nuclei. Determine the relation between k and τ.

(ii) In the population growth model, find an expression for the time for the population to double in size. (8)

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May 2026

B.Sc. Mathematics/B.Sc. Mathematics & Computing

II SEMESTER

Differential Equations (BMH–202A)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Prove that $y = 2\sin x + 3\cos x$ is a solution of the differential equation :

$$\frac{d^2y}{dx^2} + y = 0. \quad (1.5)$$

(b) Find the order and degree of the following differential equation:

$$\frac{d^3y}{dx^3} + 5\frac{dy}{dx} + 4 = 0. \quad (1.5)$$

(c) Solve the differential equation :

$$y = px + p(1-p), \text{ where } p = \frac{dy}{dx}. \quad (1.5)$$

(d) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0. \quad (1.5)$$

(e) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0. \quad (1.5)$$

(f) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = 0. \quad (1.5)$$

(g) Define a homogeneous differential equation with constant coefficients. (1.5)

(h) Write a short note on radioactivity. (1.5)

(i) State the necessary and sufficient condition for the differential equation :

$M(x, y)dx + N(x, y)dy = 0$ to be exact. (1.5)

(j) Consider the differential equation :

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0.$$

Its solutions are e^x, e^{-x} and e^{2x}. Determine whether they are linearly independent or not. (1.5)

PART–B

2. (a) Solve the initial-value problem:

$$\frac{dy}{dx} = -\frac{x}{y}, \; y(3) = 4. \quad (7)$$

(b) Solve the differential equation :

$$(x^2 - 3y^2)dx + 2xy\,dy = 0. \quad (8)$$

3. (a) Solve the initial-value problem :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, \; y(0) = -3, \; y'(0) = -1. \quad (7)$$

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} + y = \tan x. \quad (8)$$

4. (a) Solve the differential equation :

$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 3x^2 + 4\sin x - 2\cos x. \quad (7)$$

(b) Solve the differential equation:

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3. \quad (8)$$