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May-2026

B.Sc. Mathematics/MAC - II SEMESTER

Real Analysis (BMH - 201 A)

Max. Marks:75

Time: 3 Hours

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Explain completeness property of real numbers. (1.5)

(b) Differentiate between algebraic properties and order properties of real numbers. (1.5)

(c) Define the following terms: (1.5)

1. Bounded set
2. Supremum
3. Infimum

(d) Prove that a sequence in **R** can have at most one limit. (1.5)

(e) Explain the followings: (1.5)

1. Bounded sequence
2. Increasing Sequence
3. Decreasing Sequence

(f) State Monotone Subsequence Theorem. (1.5)

(g) Explain Divergence criteria. (1.5)

(h) What do you understand by Cauchy sequence? Explain. (1.5)

(i) Differentiate between absolute convergence and conditional convergence. (1.5)

(j) Test the convergence of the following series: (1.5)

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

PART -B

Q2 (a) State and prove Density Theorem. (8)

(b) State and prove Archimedean Property. (7)

Q3 (a) Prove that every convergent sequence of real numbers is bounded. (5)

(b) Let $X = <x_n>$ and $Y = <y_n>$ be sequences of real numbers that converges to x and y respectively. Then prove that the sequence (5)

$$X.Y = <x_n \cdot y_n>$$

converges to $x.y$

(c) State and prove Squeeze Theorem. (5)

Q4 (a) State and prove Bolzano – Weierstrass Theorem for sequences. (8)

(b) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy Sequence. (7)

Q5 (a) Explain the following test: (8)

1. Comparison Test
2. P – series Test
3. D'Alembert's Ratio Test
4. Cauchy's n^{th} Root Test

(b) Test the convergence of the following series: (7)

1. $\sum_{n=1}^{\infty} \frac{1}{n!}$
2. $\sum_{n=1}^{\infty} \frac{1}{(n-2)(n+1)}$
3. $\sum_{n=1}^{\infty} x\, e^{-x^2}$
4. $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$

Q6 (a) State and prove Monotone Convergence Theorem. (8)

(b) *If* $a, b \in R$, then show that (7)

1. $|a+b| \leq |a|+|b|$
2. $|a-b| \leq |a|+|b|$
3. $||a|-|b|| \leq |a-b|$

Q7 (a) Prove that, a monotone sequence of real numbers is properly divergent if and only if it is unbounded and (8)

1. If $X = \langle x_n \rangle$ is an unbounded increasing sequence then $Lim\,(x_n) = +\infty$
2. If $X = \langle x_n \rangle$ is an unbounded decreasing sequence then $Lim\,(x_n) = -\infty$

(b) State and prove Integral Test for series. (7)