7. (a) Evaluate the residues of :

$$f(z)=\frac{z^2}{(z-1)(z-2)(z-3)}$$

at 1, 2 and 3. (7)

(b) Evaluate :

$$\int_0^{2\pi}\frac{d\theta}{a+b\cos\theta}.$$ (8)

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May 2026

B.Sc. Mathematics/B.Sc. Mathematics & Computing–VIth Semester

Complex Analysis (BMH–601A)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any **four** questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Determine the polar form of the complex number $z = -5 + 5i$. (1.5)

(b) If $z = x + iy$, then find $z\bar{z}$. (1.5)

(c) Determine the domain and range of the function :

$f(z) = e^z$. (1.5)

(d) State the Cauchy-Riemann equations for an analytic function. (1.5)

(e) If C is the closed contour $|z| = 5$, then find $\int_c z^2 dz$. (1.5)

(f) State Cauchy's theorem. (1.5)

(g) Define an analytic function. Give an example. (1.5)

(h) Define a singular point of a function. Give an example. (1.5)

(i) Determine the number of zeroes of the function :

$$f(z) = \sin\left(\frac{1}{z}\right).$$ (1.5)

(j) Determine the period of the function $f(z) = e^z$. (1.5)

PART–B

2. (a) If the complex numbers :

$$\sin x + i \cos 2x \text{ and } \cos x - i \sin 2x$$

are complex conjugate to each other, then find the value of x. (7)

(b) Find the analytic function $f(z) = u + iv$ of which the real part is $u = e^x(x \cos y - y \sin y)$. (8)

3. (a) Prove that the function :

$$f(z) = xy + iy$$

is everywhere continuous but not analytic. (7)

(b) Integrate $f(z) = e^{iz}$ along the boundary of the triangle with vertices at the points $1 + i, -1 + i$ and $-1 - i$. Also, verify Cauchy's theorem. (8)

4. (a) Evaluate :

$$\int_C \frac{e^{2z}}{(z+1)^2} dz, \text{ where C is } |z| = 3.$$ (7)

(b) State and prove Liouville's theorem. (8)

5. (a) Find the type of singularity of the following function at $z = 0$:

$$f(z) = \frac{z - \sin z}{z^3}.$$ (7)

(b) Obtain the Taylor's and Laurent's series which represents the function :

$$f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$$

in the regions :

(i) $|z| < 2$.

(ii) $2 < |z| < 3$. (8)

6. (a) If a and b are real numbers between 0 and 1 such that:

$$z_1 = a + i,\ z_2 = 1 + ib \text{ and } z_3 = 0$$

form an equilateral triangle, then determine the values of a and b. (7)

(b) If $f(z) = u + iv$ is an analytic function of $z = x + iy$, then prove that u and v are harmonic functions. (8)