5. (a) Using the Cauchy integral formula (or its extension for derivatives), evaluate the following integrals where all contours are traversed counter clockwise :

(i) $\oint_{|z|=3} \frac{\cos(\pi z)}{(z^2-1)} dz.$

(ii) $\oint_{|z-i|} \frac{(z^2 \mid 3z \mid 2)}{(z^2+1)^2} dz.$ (10)

(b) State and prove the Cauchy integral tormula. (5)

6. (a) Find the Taylor series of $f(z) = \frac{z}{(z-1)(z-3)}$ about $z_0 = 0$ and determine its radius of convergence. (10)

(b) Prove that the zeros of a non-constant analytic function are isolated. Hence, or otherwise, show that if f and g are analytic on a domain D and $f(z) = g(z)$ on a set having a limit point in D, then $f \equiv g$ on D. (5)

7. (a) Using contour integration and the Residue Theorem, evaluate :

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$$

Clearly describe the contour used, justify why the integral over the semicircular arc vanishes in the limit, and identify all relevant poles and their residues. (10)

(b) Using the residue theorem, evaluate

$$\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$$ (5)

Roll No. ..... Total Pages : 4

323601

May-2026

B.Sc. (Maths/Maths & Comp.)- VI- SEMESTER

Complex Analysis (BMH24-601/MTU-302-V)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Let $z_1 = 3 + 4i$ and $z_2 = 1 - 2i$. Verify that $|z_1 z_2| = |z_1||z_2|$ and find $|z_1/z_2|$. (1.5)

(b) Determine all points in the complex plane where the function $f(z) = |z|^2$ is differentiable. Is it analytic anywhere? Justify briefly. (1.5)

(c) If $u(x, y) = e^x (x \cos y - y \sin y)$, verify that u is harmonic and find the harmonic conjugate v at the point (0, 0) given $v(0, 0) = 0$. (1.5)

(d) Find the image of the circle $|z| = 2$ under the mapping $w = 1/z$. (1.5)

(e) Evaluate $\int_C \bar{z}\, dz$, where C is the straight line segment from $z = 0$ to $z = 1 + i$. (1.5)

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