Roll No. ..... Total Pages : 3

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May-2026

B.Sc- Mathematics and Computing - VI SEMESTER

Tensor Tools for Mathematical Modelling

(BMH24-606/MTU-316-V)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Define a tensor of type (p, q) and explain the meaning of contravariant and covariant indices. (1.5)

(b) State the transformation law for a mixed tensor of type (1,1). (1.5)

(c) Show how the Kronecker delta δ_{ij} acts as an identity tensor under contraction. (1.5)

(d) What condition on strain ensures pure volumetric deformation? (1.5)

(e) Evaluate $\varepsilon_{ijk}\,\varepsilon_{imk}$. (1.5)

(f) Define an isotropic tensor of rank two. (1.5)

(g) Write $(\nabla \times A)_i$ and $\nabla \cdot A$ in index notation. (1.5)

(h) Define the deviatoric part of a second order tensor. (1.5)

(i) Define strain invariants. (1.5)

(j) State Cauchy's formula for the stress vector. (1.5)

PART–B

2. (a) Let A_{ij} be a second order tensor and $x'_p = a_{pi}\, x_i$ an orthogonal transformation. Starting from invariance of scalar product, rigorously derive the transformation law

$$A'_{p\varphi} = a_{pj}\, a_{\varphi j}\, A_{ij}.$$

Hence prove that the trace tr(A) and determinant det(A) are invariants under orthogonal transformations. (10)

(b) Given A = [2 1 0 1 3 0 0 0 4], verify explicitly that its trace remains unchanged under rotation about the z-axis through 90°. (5)

3. (a) Using ε_{ijk} and δ_{ij}, derive the identity

$$\varepsilon_{ijk}\ \varepsilon_{imk} = \delta_{ij}\,\delta_{jm} - \delta_{im}\,\delta_{ij}. \quad (10)$$

Using this result, derive in index notation $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$, and interpret each term physically.

(b) For vector field $A = (xy,\ y\,z,\ z\,x)$, compute $\nabla \times (\nabla \times A)$. (5)

4. (a) Let A_{ij} be a real symmetric second order tensor. Prove that its eigenvalues are real and eigenvectors corresponding to distinct eigenvalues are orthogonal. Hence obtain its spectral decomposition. (10)

(b) Find the principal values and principal directions of A = [5 2 0 2 5 0 0 0 3]. (5)

5. (a) Derive the expression for the Laplacian of a vector field in tensor notation. Show that (10)

$$\nabla^2(\nabla\varphi) = \nabla(\nabla^2\varphi).$$

(b) Given $\varphi = x^2y + yz^2$, compute $\nabla^2\varphi$. (5)

6. (a) Starting from the deformation gradient $F_{ij} = \delta_{ij} + U_{ij}$, derive the Green strain tensor and show that for small deformations it reduces to the infinitesimal strain tensor. Discuss its invariants. (10)

(b) For F = diag(2, 1, 1/2), compute the Green strain tensor and its first invariant. (5)

7. (a) Prove that any second order isotropic tensor in three dimensions is proportional to δ_{ij}. (10)

(b) For A = [3 1 0 1 3 0 0 0 2], compute I_1 and I_2 and verify the Cayley-Hamilton theorem. (5)