6. (a) Discuss the shape and geometric properties of an elliptic paraboloid :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z.$$

Show that its sections parallel to coordinate planes are parabolas and ellipses. (8)

(b) Discuss in detail the reduction of a general second-degree equation in three variables to its canonical form. Explain the method and its geometric significance. (7)

7. (a) Find the equation of the enveloping cone of the conicoid:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

with vertex at the origin. (8)

(b) Reduce the general second-degree equation :

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 1$$

to its canonical form using rotation of axes and hence classify the conicoid. (7)

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323408

May 2026

B.SC. (MATHS/MATHS & COMP.) IV SEMESTER

Analytical Geometry (BMH–401–A)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any **four** questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Define pole and polar with respect to a conic. Find the polar of the point (x_1, y_1) with respect to the circle $x^2 + y^2 = a^2$. (1.5)

(b) Write the equation of the chord of contact of tangents drawn from an external point (x_1, y_1) to the ellipse :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$ (1.5)

(c) Write the general equation of a second-degree curve and state the condition under which it represents a parabola. (1.5)

(d) Find the condition for two spheres to intersect orthogonally. (1.5)

(e) Write the equation of a right circular cone with vertex at origin and axis along the z-axis. (1.5)

(f) Find the equation of the plane section of the sphere :

$$x^2 + y^2 + z^2 = a^2$$

cut by the plane $x + y + z = a$. (1.5)

(g) State the condition for the plane $ix + my + nz = p$ to be tangent to the conicoid $ax^2 + by^2 + cz^2 = 1$. (1.5)

(h) What is an enveloping cone of a conicoid ? Write its equation when the vertex is at the origin. (1.5)

(i) What is a hyperbolic paraboloid ? Write its standard equation and mention its geometric nature. (1.5)

(j) Find the nature of the plane section of the cone :

$$x^2 + y^2 = z^2$$

when cut by the plane $z = k$. (1.5)

PART–B

2. (a) Find the polar equations of tangent and normal to the conic :

$$r = \frac{l}{1 + e\cos\theta}$$

at a given point and discuss their geometric interpretation. (8)

(b) Find the equation of the chord of contact of tangents drawn from (x_1, y_1) to the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$ (7)

3. (a) Find the equation of the plane section of a sphere:

$x^2 + y^2 + z^2 = 2ux + 2vy + 2wz + d = 0$ cut by the plane $lx + my + nz = p$. Show that the section is a circle and find its center and radius. (8)

(b) Find the equation of the circle of intersection of two spheres and show that it lies in a plane. Determine the equation of this plane. (7)

4. (a) Find the equation of a right circular cone with vertex at origin and axis along a given direction. Discuss the condition for a point to lie on it. (8)

(b) Find the equation of the tangent plane and normal line to the ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at the point (x_1, y_1, z_1). (7)

5. (a) Define the plane of contact of tangents drawn from a point to a conicoid. Find its equation for the ellipsoid and discuss its properties. (8)

(b) Reduce the second-degree equation :

$$3x^2 + 3y^2 + 3z^2 + 2xy + 2yz + 2zx = 1$$

to principal axes and identify the conicoid. (7)