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May-2026

B.Sc. Mathematics/MAC IV SEMESTER

Ring Theory and Linear Algebra (BMH-402 A)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define Ring with example. (1.5)

(b) Explain the followings: (1.5)

1. Zero Divisor
2. Integral Domain
3. Division Ring

(c) Differentiate between Prime Ideal and Maximal Ideal. (1.5)

(d) State second isomorphism theorem for rings. (1.5)

(e) Let A be a ring homomorphism from a ring R to ring S. Then prove that Ker A is an ideal of ring R. (1.5)

(f) Define vector space with example. (1.5)

(g) What do you understand by Standard Ordered Basis of vector space? Explain. (1.5)

(h) Differentiate between Linearly independent vectors and Linearly Dependent vectors. (1.5)

(i) Explain the followings: (1.5)

1. Linear Transformation
2. Linear Operator
3. Zero Transformation

(j) Let T be a linear operator on a finite dimensional vector space V over the field F. Then prove that N(T) is invariant subspace of V. (1.5)

PART -B

Q2 (a) Let R be a commutative ring with unit 1 and M be a proper ideal of ring R. (8)

Then show that R / M is a field if and only if M is maximal ideal.

(b) Prove that centre of a division ring is a field. (7)

Q3 (a) State and prove first isomorphism theorem. (8)

(b) Consider the ring homomorphism: (7)

$$\emptyset : R \rightarrow S$$

If $\emptyset$ is an isomorphism then show that $\emptyset^{-1}$ is also an isomorphism.

Q4 (a) Show that : (8)

$$V = R^n = \{(x_1, x_2, - - - -, x_n) : x_i \in R, \quad 0 \leq i \leq n\}$$

forms a vector space over the field of real numbers.

(b) State and prove Replacement Theorem. (7)

Q5 (a) State and prove Rank – Nullity theorem. (8)

(b) Prove that, (7)

$$\{ 1, \quad \sqrt{3}, \quad \sqrt{5}, \quad \sqrt{15} \}$$

Is linearly independent in R over the field of rational numbers.

Q6 (a) State and prove Third Isomorphism Theorem. (8)

(b) Let R be a commutative ring with unit 1 and I be a proper ideal of ring R. Then show that R / I is an integral domain if and only if I is prime ideal. (7)

Q7 (a) Suppose that: (8)

$$T : V \rightarrow U$$

Be a linear transformation. Then show that:

1. N(T) is a subspace of V
2. R(T) is a subspace of U

(b) Let $\beta = \{v_1, v_2, - - - -, v_n\}$ be a subset of a vector space V over the field F. Then show that, β forms a basis for V if and only if every element of V can be expressed uniquely as a linear combination of elements $v_1, v_2, - - - -, v_n$. (7)
