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Total Pages : 4

**305304**

**Dec., 2018**

**B.Sc. Ist Semester**

**MATHEMATICS III**

**(BS-301)**

Time : 3 Hours]

[Max. Marks : 75

*Instructions :*

*(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*

*(ii) Answer any four questions from Part-B in detail.*

*(iii) Different sub-parts of a question are to be attempted adjacent to each other.*

## PART–A

1. (a) Define trigonometric polynomial. (1.5)

(b) Find the Inverse Laplace transform of $\frac{1}{(s+a)^2}$. (1.5)

(c) If $L\{f(t)\} = F(s)$ then show that $L\left\{\frac{1}{t} f(t)\right\} = \int_s^{\infty} F(s)\, ds$, provided the integral exist. (1.5)

(d) State and prove change of scale property of Fourier transform. (1.5)

(e) Find the Fourier Transform of

$$f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$ (1.5)

(f) Show that $z\left(\frac{1}{n}\right) = z \log\left(\frac{z}{z-1}\right)$. (1.5)

(g) State Convolution theorem in Z-transform. (1.5)

(h) State Stoke's theorem. (1.5)

(i) Prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$. (1.5)

(j) If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, evaluate $\int_C \vec{F} \cdot dr$ along the curve C in the $xy$-plane $y = x^3$ from the point (1,1) to (2,8). (1.5)

## PART–B

2. (a) Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$, by Laplace transform. (7)

(b) Prove that $\int_{-1}^{1} \frac{T_m(x)\,T_n(x)}{\sqrt{1-x^2}}\,dx = \begin{cases} 0, & m \neq n \\ \frac{\pi}{2}, & m = n \neq 0 \\ \pi, & m = n = 0 \end{cases}$

where $T_n(x)$ is the Chebyshev's polynomial. (8)

3. (a) Apply Convolution theorem to evaluate

$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. (7)

(b) Solve $ty'' + y' + 4ty = 0$, where $y(0) = 3$, $y'(0) = 0$. (8)

4. (a) Find the Fourier Cosine transform of $e^{-ax}$, hence evaluate $\int_0^{\infty} \frac{\cos \lambda x}{x^2 + a^2}\,dx$. (7)

(b) Using finite Fourier transform, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that $u(0, t) = 0$, and $u(x, 0) = 2x$ when $0 < x < 4$, $t > 0$. (8)

5. (a) Find the Inverse Z-transform of $\frac{5z}{(2-z)(3z-1)}$. (7)

(b) Solve $y_{n+2} - 6y_{n+1} + 8y_n = 2^n + 6n$ by Z-transform. (8)

6. (a) Prove that

$\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + 3xyz^2\hat{k}$ is irrotational and find its scalar potential. (7)

(b) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a}$ be a constant vector, find the value of $\operatorname{div}\left(\dfrac{\vec{a} \times \vec{r}}{r^n}\right)$. (8)

7. (a) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$.

(b) Use the divergence theorem to evaluate $\iint_S \vec{F} . \hat{n} dS$ where $\vec{F} = x^3\hat{i} + x^2 y\hat{j} + x^2 z\hat{k}$ and S is the surface bounding of the region $x^2 + y^2 = a^2$, $z = a$, $z = b$. (8)

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