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May 2026

B.Sc. Mathematics/MAC - IV SEMESTER

Ring Theory and Linear Algebra-I (BMH24-403)

Time: 3 Hours

Max. Marks:75

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Show that Boolean ring R is commutative. (1.5)

(b) Explain the Ring of Quaternions. (1.5)

(c) Prove that the set of even integers forms an ideal of ring of integers $(\mathbf{Z}, +, .)$. (1.5)

(d) Show that $2\mathbf{Z}$ and $3\mathbf{Z}$ are maximal ideals in $\mathbf{Z}$. (1.5)

(e) Show that the mapping is a ring homomorphism from C, the set of complex numbers onto C. (1.5)

$$a + i\,b \rightarrow \quad a - i\,b$$

(f) If S is any subset of a vector space V over the field F. Then show that 'span(S)' is a smallest subspace of vector space V containing S over the field F. (1.5)

(g) Show that $Symm\,(M_n(F))$ forms a subspace of $M_n(F)$. (1.5)

(h) Check whether the following set of vectors are linearly independent or dependent. (1.5)

$$A = \{(0, 1, -2), (1, -1, 1), (1, 2, 1)\}$$

(i) Differentiate between Linear Transformation and Linear Operator. (1.5)

(j) Consider the linear transformation (1.5)

$$T: R^3 \rightarrow R^2 \quad defined\ by \quad T(x,y,z) = (x, \quad y - 3z)$$

Find N (T) and R (T).

PART -B

Q2 (a) Let I be a two – sided ideal in a ring R. Then show that R / I is a ring under the coset addition and multiplication (8)

(b) Let (7)

$$R = Z \oplus Z \oplus Z$$

And let

$$S = \{(a,b,c) \in R \ / \ a + c = b\}$$

Prove or disprove that S is a subring of R.

Q3 (a) State and prove Third Isomorphism Theorem. (8)

(b) Prove that, if F is a field of characteristic P, then F contains a subfield isomorphic to Z_p and if F is a field of characteristic zero, then F contains a subfield isomorphic to the field of rational numbers. (7)

Q4 (a) Let (x_1, x_2) and (y_1, y_2) be two vectors in R^2. Then show that: (8)

1. $x_1 y_2 = x_2 y_1$ if and only if (x_1, x_2) and (y_1, y_2) are linearly dependent.
2. If $T = \{(x_1, x_2), (y_1, y_2)\}$ is linearly independent set, then T is basis of R^2

(b) Let T be a finite subset of a vector space V over the field F such that, (7)

$$V = Span(T)$$

Then show that, there exists a subset of T which is a basis of V.

Q5 (a) Let $T : V \to U$ be a linear transformation, where V and U are finite dimensional vector spaces over the same field. Then prove that the following statements are equivalent: (8)

1. T is one – to – one
2. T is onto
3. Rank (T) = Dim (V)

(b) Let V be a finite dimensional vector space with an ordered basis A and let T be a linear operator on a vector space V. Then prove that T is invertible if and only if $[T]_A$ is invertible and (7)

$$[T^{-1}]_A = ([T]_A)^{-1}$$

Q6 (a) Prove that every field is an integral domain. Is converse true? Justify your answer with suitable example. (8)

(b) Prove or disprove that the field of real numbers is ring – isomorphic to the field of complex numbers. (7)

Q7 (a) Let V and U be vector spaces over a field F and let

$$T, S : V \rightarrow U$$

Be a linear transformations. Then show that:

1. aT + S is linear for all $a \in F$
2. L (V, U) forms a vector space over F (8)

(b) Let S be a finite subset of a vector space V containing N vectors such that Span (S) = V and let T be a linearly independent subset of V with L vectors. Then prove that $L \leq N$ and there exists a subset U of S containing (N – L) vectors such that

V = Span (T ∪ U) (7)
