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May 2026

B.Sc. MATHEMATICS/B.SC. MATHEMATICS & COMPUTING IV SEMESTER

Theory of Real Functions (BMH24–402)

Time : 3 Hours]

[Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any **four** questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find the cluster points of the set of natural numbers $\mathbb{N}$. (1.5)

   (b) State the Squeeze Theorem. (1.5)

   (c) State the sequential criterion for continuity. (1.5)

   (d) State the Maximum-Minimum Theorem. (1.5)

   (e) Define a uniformly continuous function. (1.5)

(f) Find the derivative of

$$f(x)=\frac{x}{1+x^2}. \qquad (1.5)$$

(g) State Rolle's Theorem. (1.5)

(h) State Darboux's Theorem. (1.5)

(i) Define a convex function. (1.5)

(j) Find:

$$\lim_{x\to 2}\left(x^2+1\right)\left(x^3-4\right). \qquad (1.5)$$

PART–B

2. (a) Let $A\subseteq\mathbb{R}$ and c be a cluster point of A. Let $f:A\to\mathbb{R}$ be a function. Prove that f can have only one limit at c. (7)

(b) (i) Prove that:

$$\lim_{x\to 0}\left(\frac{\cos x-1}{x}\right)=0.$$

(ii) Prove that:

$$\lim_{x\to 0}\ \sin x=0. \qquad (8)$$

3. (a) State and prove Bolzano's Intermediate Value Theorem. (7)

(b) (i) Prove that $f(x)=\frac{1}{x}$ is not continuous at x = 0.

(ii) Prove that g(x) = sin x is continuous on $\mathbb{R}$. (8)

4. (a) Prove that a function f is uniformly continuous on the interval (a, b) if and only if it can be defined at the endpoints a and b such that the extended function is continuous on [a, b]. (7)

(b) Let $I\subseteq\mathbb{R}$ be an interval and $c\in I$. Let $f:I\to\mathbb{R}$ and $g:I\to\mathbb{R}$ be functions that are differentiable at c. Prove that fg is differentiable at c and:

$$(fg)'(c)=f'(c)g(c)+f(c)g'(c). \qquad (8)$$

5. (a) State and prove Cauchy's Mean Value Theorem. (7)

(b) Prove that $e^x\geq 1+x$ for all $x\in\mathbb{R}$. (8)

6. (a) Prove that :

$$\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

does not exist in $\mathbb{R}$. (7)

(b) Let I be a closed bounded interval. Let $f:I\to\mathbb{R}$ be continuous on I. Prove that the set $f(I)=\{f(x):x'\in I\}$ is a closed bounded interval. (8)

7. (a) Let $f:A\to\mathbb{R}$ be a uniformly continuous function on a subset A of $\mathbb{R}$ and $\{x_n\}$ be a Cauchy sequence in A. Prove that $\{f(x_n)\}$ is a Cauchy sequence in $\mathbb{R}$. (7)

(b) State and prove Taylor's theorem. (8)