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May 2026

B.Sc. Mathematics MAC 2nd Semester

Calculus-II (BMH23-201)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) State first fundamental theorem of Calculus (1.5)

(b) Define logarithmic function and exponential function. (1.5)

(c) Write the equation of Parabolas with vertex (h, k) and axis parallel to y-axis. (1.5)

(d) If $y = f(x)$ is a smooth curve on the interval $[a, b]$, write the formula for finding the arc length L of this curve over $[a, b]$. (1.5)

(e) Sketch the graph of the parabola $y^2 - 6x = 0$.. (1.5)

(f) Find an equation for the ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$. (1.5)

(g) Evaluate $\int_0^1 (3x + 2)^{3/2}\, dx$. (1.5)

(h) Find N(t) for the circular helix x=a cost, y=a sint, z=ct where a>0. (1.5)

(i) Evaluate $\int_{-\pi/2}^{\pi/2} cos^7 x dx$. (1.5)

(j) State Kepler's first law of planetary motion. (1.5)

PART -B

Q2 (a) Let f be continuous on $[a, b]$ and F be any antiderivative of f on $[a, b]$. Prove that (8)

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

(b) Find the reduction formula of $\int x^n \cos x\ dx$. (7)

Q3 (a) Sketch the graph in polar coordinates of $r = \frac{2}{1-\cos\theta}$. (8)

(b) Identify and sketch the curve (7)

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$

Q4 (a) Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x-axis. (8)

(b) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between x=0 and x=1 about the x-axis. (7)

Q5 (a) Let $r_1(t) = (\tan^{-1} t)\hat{\imath} + (\sin t)\hat{\jmath} + t^2\hat{k}$ and $r_2(t) = (t^2 - t)\hat{\imath} + (2t - 2)\hat{\jmath} + (\ln t)\hat{k}$. (8)

The graphs of $r_1(t)$ and $r_2(t)$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin

(b) (i) Evaluate the definite integral (7)

$$\int_0^1 (2t^2\,\hat{\imath} + 5t^3\hat{\jmath} + \hat{k})dt.$$

(ii) Find $r(t)$ given that $r'(t) = \langle 3, 2t\rangle$ and $r(1) = \langle 2, 5\rangle$.

Q6 (a) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval [0, 2] is rotated about the line y=−1. (8)

(b) Find the curvature κ(t) for the circular helix x=a cost, y=a sint, z=ct where a>0. (7)

Q7 (a) Let $r(t)$ be a smooth vector-valued function in 2-space or 3-space, then for each value of t at which $T'(t)$ and $r''(t)$ exist, show that the curvature κ can be expressed as (8)

(i) $\kappa(t) = \frac{\|T'(t)\|}{\|r'(t)\|}$,

(ii) $\kappa(t) = \frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|^3}$.

(b) State and prove Kepler's second law of planetary motion. (7)