The ends of bar are kept at zero temperature and initial temperature is given by :

$f(x) = 10 - x.$

Find the temperature distribution in bar. (10)

(b) A string is stretched and fastened to two points distances l apart. Find the displacement $y(x, t)$ at any point at a distance x from one end at a time t, given that :

$$y(x,0) = A\sin\frac{2\pi x}{l}. \quad (5)$$

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May 2026

B.Sc. (Maths/Maths & Comp.) IV Semester

Partial Differential Equations (MTU–208–V)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any **four** questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Define the order and degree of a partial differential equation. (1.5)

   (b) Solve $p \tan x + q \tan y = \tan z$. (1.5)

   (c) What is a non-homogeneous linear partial differential equation? (1.5)

   (d) Find the solution of the P.D.E: $(D^2 + D'^2)z = 0$. (1.5)

   (e) What is meant by reducible partial differential equations with variable coefficients? (1.5)

   (f) What is an elliptic partial differential equation? (1.5)

(g) State the discriminant used for classifying second-order linear PDEs. (1.5)

(h) Determine the characteristic of the equation :

$$e^{2x} \cdot r + 2e^{x+y} \cdot s + e^{2y} \cdot t = 0. \quad (1.5)$$

(i) Write the standard form of Laplace's equation in Cartesian coordinates. (1.5)

(j) What is the standard form of the one-dimensional wave equation? (1.5)

PART–B

2. (a) Find the differential equation of all spheres of fixed radius having centre in xy-plane. (7)

(b) Solve the following partial differential equation :

$$z(xy + z^2)(px - qy) = x^4. \quad (8)$$

3. (a) Show that the equations :

$$xp - yq = x, \; x^2p + q = xz$$

are compatible and find their solution. (8)

(b) Solve the following partial differential equation:

$$r - 3s + 2t = e^{2x+3y} + \sin(x - 2y),$$

where the symbols have their usual meanings. (7)

4. (a) Solve the PDE :

$$\left(D^2 - D'^2 - 3D + 3D'\right)z = xy + e^{x+2y}. \quad (7)$$

(b) Solve the following partial differential equation:

$$\left(x^2D^2 + 2xyDD' + y^2D'^2\right)z = x^my^n. \quad (8)$$

5. (a) Show that the partial differential equation :

$$z_{xx} + 2xz_{xy} + (1 - y^2)z_{yy} = 0$$

is elliptic for all values of x and y in the region :

$$x^2 + y^2 < 1,$$

parabolic on the boundary and hyperbolic outside this region. (8)

(b) Classify the equation :

$$y^2u_{xx} - 2xy\, u_{xy} + x^2 u_{yy} = 0$$

and reduce it to its canonical form. (7)

6. (a) Classify the partial differential equation :

$$u_{tt} + tu_{xt} + xu_{xx} + 2u_t + u_x + 6u = 0. \quad (5)$$

(b) Solve the PDE $u_{xx} = u_y + 2u$ by the method of separation of variables given that :

$$u(0, y) = 0, u_x(0, y) = 1 + e^{-3y}. \quad (10)$$

7. (a) The heat flow in a bar of 10 cm of homogeneous material is governed by :

$$\text{PDE } u_t = c^2u_{xx}.$$