

Roll No.

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M. Sc. (Mathematics) (Second Semester)

Mathematical Statistics (MTP-102-V)

Time : 3 Hours

[Maximum Marks : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other. Students can use calculator. Specific tabular values are mentioned in the question paper, no more tables are required.

Part A

1. (a) A number is chosen at random among the first 120 natural numbers. What is the probability that the number chosen being a multiple of 5 or 15 ? **1.5**
- (b) For a continuous random variable $X \geq 0$, show that $E(X) \geq 0$. **1.5**

- (c) Find $\text{Var}(c)$, where c is a constant. 1.5
- (d) Determine the distribution for a random variable X if its moment generating function is given by $M_x(t) = e^{(e^t - 1)/4}$. 1.5
- (e) In a normal probability curve, area to the right of the point x_1 is 0.6 and to the left of the point x_2 is 0.7. Is $x_1 > x_2$ or $x_1 < x_2$ or $x_1 = x_2$? Explain. 1.5
- (f) Write any two assumptions of Student's t -test. 1.5
- (g) In hypothesis testing, write the test statistic for the test of significance for difference of means. 1.5
- (h) In left-tailed test, what is the critical value at 10% level of significance? 1.5
- (i) Find the mode of a random variable X if it follows Poisson distribution $X \sim P(3.4)$. 1.5
- (j) Define Bernoulli trial with example. 1.5

Part B

2. (a) Find the probability that in a random arrangement of the letters of the word "UNIVERSITY", the two I's do not come together. 7

- (b) The diameter of an electric cable, say X , is assumed to be a continuous random variable satisfying $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$. Check if $f(x)$ is a probability density function. Determine a number b such that $P(X < b) = P(X > b)$. 8

3. (a) The joint probability mass function of two random variables X_1 and X_2 is given by

$$p(x_1, x_2) = \frac{x_1 + x_2}{21}, \quad x_1 = 1, 2, 3, \quad x_2 = 1, 2.$$

Find the marginal probability mass functions for X_1 and X_2 . Check if X_1 and X_2 are independent. 7

- (b) Let X be a $B(2, p)$ variate and Y be a $B(4, p)$. If $P(X \geq 1) = \frac{5}{9}$, find $P(Y \geq 1)$. 8

4. (a) The marks obtained by a number of students in Mathematics are assumed to be approximately normally distributed with mean value 65 and with a standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70?

$$\text{Given } P(0 \leq Z \leq 1) = 0.3413. \quad 7$$

- (b) Random variables $X_i, i = 1, 2, 3, \dots$ are independent and all of them have the same distribution function defined by :

$$f(x) = \frac{1}{\sqrt{18\pi}} \exp\left[-\frac{(x-1)^2}{18}\right], -\infty < x < \infty.$$

Find the distribution of $3X_1 - X_2 + X_3$. 8

5. (a) If X follows Poisson distribution with parameter λ and μ_r denotes its r^{th} order moment with respect to mean, then prove the recurrence relation : 7

$$\mu_{r+1} = r\lambda\mu_{r-1} + \lambda \frac{d\mu_r}{d\lambda}$$

- (b) A dataset has a mean of 50 and a standard deviation of 8. What is the minimum probability that a randomly selected data point lies within 34 and 66 ? 8

6. (a) If X follows Gamma distribution with parameter λ , then find its cumulant generating function and hence find its variance. 7

- (b) On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups, the upper 30% and the remaining 70%. Consider the first

question of this examination. Among the first group, 40 had the correct answer and among the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good enough to distinguish between the ability of the two groups of candidates ? 8

7. (a) A random sample of 8 envelopes is taken from letter box of a post office and their weights in grams are found to be 12.1, 11.9, 12.4, 12.3, 11.9, 12.1, 12.4, 12.1 and the sample variance is 0.04. Using Student's t -test, check if this sample at 1% level indicates that the average weight of envelopes received at this post office is 12.35 grams. (Tabulated value of t for 7 degrees of freedom at 1% level is 3.499). 7

- (b) A die is thrown 60 times with the following results :

Face : 1 2 3 4 5 6

Frequency : 8 7 12 8 14 11

Test at 5% level of significance if the die is unbiased, assuming that $P(\chi^2 > 11.1) = 0.05$ with 5 d.f. 8

