(b) Let $T : R^3 \to R^2$ be a linear transformation such that $T(x, y, z) = (x - y, 2z)$. Find the rank and nullity of T. Also, find whether T is 1-1 or onto. (7)

6. (a) Let $T : R^2 \to R^3$ be a linear transformation defined by $T(x, y) = (2x - y, 3x + 4y, x)$. Compute the matrix of T in the standard bases of R^2 and R^3. (8)

(b) Test the consistency of the system and solve, if consistent:

$$x + 2y - z - 5w = 4$$
$$x + 3y - 2z - 7w = 5$$
$$2x - y + 3z = 3 \quad (7)$$

7. (a) Use Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{(1, 1, 0), (0, 1, 0), (1, 0, 1)\}$ of R^3 with the standard inner product. (8)

(b) Let $V = R^2 = \{(a_1, a_2) : a_1, a_2 \in R\}$. Test the following subsets are subspaces of V.

(i) $W_1 = \{(a, 0) \mid a \in R\}$

(ii) $W_2 = \{(0, a) \mid a \in R\}$

(iii) $W_3 = \{(x, y) \mid 2x + 3y = 0; x, y \in R\}$

(iv) $W_4 = \{(x, y) \mid 2x + 3y = 2; x, y \in R\}$. (7)

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321205

May-2026

B.Sc. (Physics) - II SEMESTER

Linear Algebra

(OSU–112–V)

Time : 3 Hours] [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART – A

1. (a) Let $T : R^2 \to R^3$ is a linear transformation such that $T(x, y) = (2x - y, x - y, 2x)$, $\forall (x, y) \in R^2$. Is T one-to-one? (1.5)

(b) If the vector $(1, -2, k)$ in R^3 is a linear combination of the vectors $(2, -1, -5)$ and $(3, 0, -2)$, then find the value of k. (1.5)

(c) Consider the vector space $P_2(R)$ of polynomials of degree at most 2 with real coefficients. Find a basis for this space and determine its dimension. (1.5)

(d) Find the eigenvalues of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. (1.5)

(e) Check if the transformation $T : R^2 \to R^3$ defined by $T(x, y) = (2y, x - y, x + 5)$ is linear or not. (1.5)

(f) Define the basis of a vector space. Give two examples. (1.5)

(g) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 4 & 1 & 10 \end{bmatrix}$. (1.5)

(h) Is $W = \{(x, y, z) \mid x - 5y + 4z - 9 = 0\}$ a subspace of R^3? Justify your answer. (1.5)

(i) Test whether the set of vectors (1, 0, –1), (2, 5, 1), (0, –4, 3) in R^3 is linearly independent. (1.5)

(j) The vector space of all 2×2 real matrices is denoted by $M_{2\times2}(R)$. Determine whether the set of matrices

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\right\}$$

is linearly dependent or independent in $M_{2\times2}(R)$. (1.5)

PART – B

2. (a) Let $T : R^3 \to R^3$ is a linear transformation such that $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$. Prove that T is invertible and find the formula for T^{-1}. (8)

(b) If $\alpha = (4, 3, 5)$, $\beta = (0, 1, 3)$, $\gamma = (2, 1, 1)$, $\delta = (4, 2, 2)$ in R^3 then prove that

(i) α is a linear combination of β and γ.

(ii) β is not a linear combination of γ and δ. (7)

3. (a) Investigate for what values of β and μ the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \beta z = \mu$$

have (i) no solution (ii) unique solution (iii) infinite number of solutions. (8)

(b) If $V = \{(a_1, a_2) : a_1, a_2 \in R\}$ and addition and scalar multiplication are defined as $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $\alpha(a_1, a_2) = (a_1, 0)$. Test V is a vector space over R? (7)

4. (a) Find the dimension of the subspace spanned by (1, 1, 2, 4), (2, –1, –5, 2), (1, –1, –4, 0) and (2, 1, 1, 6) in R^4. (8)

(b) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$ (7)

5. (a) Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 3 & 1 & 1 & 3 \end{bmatrix}$ by reducing to echelon form. (8)