5. (a) Prove that continuous image of a compact metric space is compact. 8

(b) Prove that every closed subset of a compact metric space is compact. 7

6. (a) Prove that Cartesian product of a non-empty family of non-empty sets is non-empty. 8

(b) Let (R, d) be the usual metric space. Find the derived set of the following : 7

(i) $A = \left\{\frac{1}{n} : n \in N\right\}$

(ii) $[0, 1)$

(iii) Natural numbers N

(iv) Rational numbers Q.

7. (a) State and prove Banach Contraction Principle. 8

(b) Prove that a metric space is sequentially compact if and only if every infinite subset has a limit point. 7

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May 2026

B.Sc. (Mathematics)/B.Sc. (Mathematics & Computing) (Sixth Semester)

Set Theory and Metric Space (BMH-602A)

Time : 3 Hours] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) State Zron's Lemma. 1.5

(b) Explain Partially ordered set with example. 1.5

(c) State Schroder-Bernstein Theorem. 1.5

(d) Show that in a discrete metric space X, every subset of X is open. **1.5**

(e) Explain the following : **1.5**

(i) Close set

(ii) Isolated points

(iii) Boundary of a set

(f) State Cantor's Intersection theorem. **1.5**

(g) Explain the following : **1.5**

(i) Dense set

(ii) Homeomorphism

(iii) Separable space.

(h) Give an example of a collection of subsets having Finite Intersection Property. **1.5**

(i) State Heine Borel Theorem. **1.5**

(j) Explain Bolzano Weierstrass Property. **1.5**

Part B

2. (a) State and prove Cantor's Theorem for sets. **8**

(b) Let a, 0 and 1 are cardinal numbers. Then prove the following : **7**

(i) $a + 0 = 0 + a = a$

(ii) $a.0 = 0.\ a = 0$

(iii) $a.1 = 1.a = a.$

3. (a) Prove that a subset F of a metric space (X, d) is closed if and only if F contains all its limit points. **8**

(b) Prove that in a metric space (X, d), the intersection of a finite number of open sets is open. **7**

4. (a) State and prove Baire's Category Theorem. **8**

(b) Prove that a metric space is complete if and only if every Cauchy sequence in it has a convergent subsequence. **7**