6. (a) State and prove Hilbert formula. 8

(b) Solve the integral equation : 7

$$y(x) = \sin x - \frac{1}{2\pi}\int_0^{2\pi} y(t)\cot\left(\frac{t-x}{2}dt.\right).$$

7. (a) Construct Green's function for a homogeneous boundary value problem :

$$\frac{d^4 y}{dx^4} = 0; y(0) = y'(0) = y(1) = y'(0) = 0 \text{.} \quad 8$$

(b) Reduce the boundary value problem : 7

$y'' = \lambda y + e^x; y(0) = y(1) = 0$ to an integral equation.

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Total Pages : 04

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May 2026

M.Sc. (Mathematics) (Fourth Semester)

Integral Equations (MATH21-854)

Time : 3 *Hours*] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Define fredholm integral equation of second kind. 1.5

(b) Define symmetric kernel with an example. 1.5

(c) What are Characteristic values and Characteristic functions of Homogeneous Fredholm integral equations ? 1.5

(d) Find the inverse Laplace transform of $\frac{\cos at}{a}$. 1.5

(e) State Hilbert's theorem. 1.5

(f) Using Fredholm determinants, find the resolvent kernel of : 1.5

$$K(x,t)=2x-t, 0 \le x \le 1.$$

(g) State Fredholm's first fundamental theorem. 1.5

(h) Define influence function. 1.5

(i) Define Direc-Delta function. 1.5

(j) What is the general form of the Abel singular Integral equation ? 1.5

Part B

2. (a) State and prove Poincare-Bertrand transformation formula. 8

(b) Show that $y(x) = \cos 2x$ is a solution of the integral equation $y(x)$: 7

$$y(x)=\cos x+3\int_0^{\pi} K(x,t)y(t)dt \quad \text{where}$$

$$K(x,t)=\begin{cases}\sin x\cos t, 0 \le x \le t \\ \cos x \sin t, t \le x \le \pi\end{cases}.$$

3. (a) Find the eigen values and eigen functions of Homogeneous integral equations :

$$y(x)=\lambda\int_0^{\pi}(\cos^2 x\cos 2t+\cos 3x\cos^3 t)y(t)dt. \quad 8$$

(b) Convert $y''(x)-3y'(x)+2y(x)=4\sin x$ with initial conditions $y(0)=1, y'(0)=-2$ into a Volterra integral equation of second kind. Conversely, derive the original differential equation with initial conditions from the integral equation obtained. 7

4. (a) With the aid of resolvent kernel find the solution of integral equation :

$$e^{x^2}+\int_0^x e^{x^2-t^2}y(t)dt. \quad 8$$

(b) Solve $y(x)=\cos x+\lambda\int_0^{\pi}\sin(x-t)y(t)dt$. 7

5. (a) Solve the symmetric integral equation :

$y(x)=(x+1)^2+\int_{-1}^{1}(xt+x^2t^2)y(t)dt$ by using Hilbert-Schmidt theorem. 8

(b) Find the resolvent kernel and solution of :

$$y(x)=f(x)+\lambda\int_0^1(x+t)y(t)dt. \quad 7$$