6. (a) Solve the integral equation : 5

$$f(x)=\int_b^x \frac{y(t)}{\left(t^2-x^2\right)^\alpha}dt,\ 0<\alpha<1;\ a<x<b$$

(b) Using Green's function, reduce the following boundary value problem to a Fredholm integral equation : 10

$$y''+\lambda y=x,\ y(0)=y(\pi/2)=0$$

7. Find the solution of the following Hilbert Singular integral equation of the second kind : 15

$$ay(x)=f(x)-\frac{b}{2\pi}\int_0^{*2\pi} y(t)\cot\left(\frac{t-x}{2}\right)dt$$

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May 2026

M. Sc. (Mathematics) (Fourth Semester)

Integral Equations (MTP-208-V)

Time : 3 *Hours*] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other. Notations used in the paper have their usual meanings.

Part A

1. (a) What is the Fredholm Integral Equation of the second kind ? 1.5

(b) Find the iterated kernels for the following kernel : 1.5

$$K(x,t)=\sin(x-2t),\ 0\le x\le 2\pi,\ 0\le t\le 2\pi$$

(c) Find the Laplace transform of the function $F(t) = e^{at}\sin at$. 1.5

(d) Define symmetric kernel $K(x, t)$ with an example. 1.5

(e) Define Cauchy Integral. 1.5

(f) What is the general form of the Abel's Singular Integral Equation ? 1.5

(g) How is Green's function G(x, t) used to solve differential equations ? 1.5

(h) What is the Wronskian of the functions e^x, e^{2x} and e^{3x}. 1.5

(i) What is the solution of the integral equation

$y(x) = 1 + \int_0^x y(t)\, dt$? 1.5

(j) State Hilbert formula. 1.5

Part B

2. (a) Solve the following integral equation by means of resolvent kernel : 7

$$y(x) = x + \int_0^x (t - x)\, y(t)\, dt$$

(b) Transform the following boundary value problem into an integral equation : 8

$$\frac{d^2 y}{dx^2} + \lambda y = 0 \text{ with } y(0) = 0,\ y(l) = 0$$

3. (a) Construct Green's function of the boundary value problem : 7

$$y'' = 0,\ y(0) = y(1) = 0$$

(b) Show that the integral equation : 8

$$y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\, y(t)\, dt$$

possesses no solution for $f(x) = x$, but that it possesses infinitely many solutions when $f(x) = 0$.

4. Convert the differential equation $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with initial conditions $y(0) = 1$, $y'(0) = -2$ into a Volterra integral equation of the second kind. Conversely, derive the original differential equation with initial conditions from the integral equation obtained. 15

5. (a) Solve the integral equation :

$$Y(t) = 1 + \int_0^t Y(x)\sin(t - x)\, dx$$

using Laplace transform and verify your solution. 7

(b) Using Fredholm determinants, find the resolvent kernel, when : 8

$$K(x, t) = xe^t,\ a = 0,\ b = 1$$