Jan. 2022
B.Sc. (Phy) - III SEMESTER
Mathematical Physics-II (BPH-301)

Time : 90 Minutes]　　　　　　　　　　[Max. Marks : 25

Instructions :

　1.　*It is compulsory to answer all the questions (1 mark each) of Part-A in short.*

　2.　*Answer any three questions from Part-B in detail.*

　3.　*Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1.　(a)　What is a periodic function? (1)

　　(b)　Evaluate $\Gamma\left(-\dfrac{3}{2}\right)$ (1)

　　(c)　Determine the value of a_0 for a periodic function defines as $f(x) = x - x^2;\ -\pi < x < \pi$. (1)

　　(d)　What is an odd function? (1)

　　(e)　Write the orthogonality relation for Bessel's function. (1)

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(f) Prove that $J_n(-x) = -J_n(x)$.

(1)

(g) What is generating function of Legendre's polynomial.

(1)

(h) Prove that $P_n(1) = 1$.

(1)

(i) Prove that $\Gamma\left(\dfrac{1}{2}\right) = \sqrt{\pi}$.

(1)

(j) Prove that $\beta(p, q) = \beta(p + 1, q) + \beta(p, q + 1)$. (1)

PART-B

2. (a) Obtain a Fourier series to represent the function $f(x) = |\sin x|$ for $-\pi < x < \pi$.

(3)

(b) Prove Parseval's Indentity for fourier series.

(2)

3. (a) Prove that $J_n(x) = \dfrac{1}{\pi}\int\limits_{0}^{\pi} \cos(n\theta - x \sin \theta)d\theta$, using generating function for Bessel's function.

(3)

(b) Prove that $xJ_n' = -nJ_n + xJ_{n-1}$.

(2)

4. Find the deflection $u(x, y, t)$ of the square membrane with $a = b = c = 1$, if the initial velocity is zero and the initial deflection is $f(x, y) = A \sin \pi x \sin 2\pi y$.

(5)

5. (a) Derive the Rodrigue's formula for Legendre's polynomial $P_n(x)$. (3)

   (b) Prove that $P_n'(1) = \frac{1}{2}n(n+1)$. (2)

6. (a) Express the integral $\int\limits_{a}^{b} (b-x)^{m-1} (x-a)^{n-1}\, dx$ with $b > a$, $m > 0$, $n > 0$ in terms of gamma and beta functions. (2)

   (b) Derive the relationship between gamma and beta functions. (3)

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