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# 235101

**December, 2019**

**B.Sc. Honors (Physics) 1st Semester**

**Mathematical Physics-I (BPH-101)**

Time : 3 Hours]

[Max. Marks : 75

*Instructions :*

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART–A

**1.** (a) If $\vec{A}$ and $\vec{B}$ are two unit vectors and $\theta$ is the angle between them, then find the value of $\theta$ such that $\vec{A}+\vec{B}$ is a unit vector. (1.5)

(b) Define scalar product of two vectors and $\vec{A}$ and $\vec{B}$ along with its geometrical interpretation. (1.5)

(c) Two vector fields are given as (i) $\vec{A} = m\hat{i}$ and (ii) $\vec{B} = m\vec{r}$. Find $\nabla . \vec{B}$ and $\nabla \times \vec{B}$ and draw $\vec{A}, \vec{B}, \nabla . \vec{B}$ and $\nabla \times \vec{B}$. (1.5)

(d) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$ and $y_3 = \frac{x_2 x_1}{x_3}$ then show that the Jacobian of $(y_1, y_2, y_3)$ with respect to $(x_1, x_2, x_3)$ is 4. (1.5)

(e) Find the values of integrals (i) $\int_1^5 (3x-2)\delta(x-2)\,dx$ (ii) $\int_3^5 (3x-2)\delta(x-2)\,dx$. (1.5)

(f) Plot the graph corresponding to $y = |x + 2|$. (1.5)

(g) Find the Integrating Factor (I.F.) for the 1st order linear differential equation

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x.$$ (1.5)

(h) Show that complementary function (C.F.) of the differential equation $\frac{d^2y}{dx^2} + y = x - \cot x$ is given as C.F. = $a \cos x + b \sin x$, where '$a$' and '$b$' are constants. (1.5)



(i) In spherical polar coordinates, find $\nabla f$ for scalar field $f = \frac{1}{r}$. (1.5)

(j) Prove that $\nabla \times (\phi \vec{V}) = (\nabla \phi) \times \vec{V} + \phi \nabla \times \vec{V}$, where $\phi$ is a scalar field. (1.5)

## PART - B

**2.** (a) Write the general form of a linear differential equation of 1st order and show that its solution is given asy (I.F.) = $\int$ Q(I.F.)$dx$ + $c$, where P and Q are the functions of $x$, $c$ is a constant and I.F. is the integrating factor given as I.F. = $e^{\int P dx}$. (10)

(b) Solve the differential equation

$$(x^2+1)\frac{dy}{dx} + 2xy = x^2.$$ (5)

**3.** (a) If $(\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that (i) $\nabla(r^n) = nr^{n-2}$ where $r = |\vec{r}|$ (ii) $\nabla(\vec{a}.\vec{r}) = \vec{a}$, where $\vec{a}$ is a constant vector. (5)

(b) Define gradient of a scalar field and also discuss its geometrical interpretation. (10)

4. Discuss the variation method of parameters to find the particular integral (P.I.) of differential equation

$$\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=X.$$

Also apply this method to solve the differential equation :

$$\frac{d^2y}{dx^2}+y=\tan x.$$ (15)

5. (a) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1,\ z=0,$ under field of force

$$\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}.$$

Is the force conservative? (5)

(b) State Stokes theorem and verify it for the vector field $\vec{F}=z\hat{i}+x\hat{j}+y\hat{k},$ where C is the unit circle in $x$–$y$ plane bounding the hemisphere $z=\sqrt{1-x^2-y^2}$. (10)

6. (a) Show that $\iint_S \vec{F}.\hat{n}\,ds=\frac{3}{2},$

where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and 'S' is the surface of the cube bounded by the planes $x$ = 0, $x$ = 1, $y$ = 0, $y$ = 1, $z$ = 0, $z$ = 1. (10)



(b) If $y_1=e^{-x}\cos x,\ y_2=e^{-x}\sin x$

and $\frac{d^2y}{d^2x}+2\frac{dy}{dx}+2y=0,$ then (i) Calculate the Wronskin (ii) Verify that $y_1$ and $y_2$ satisfy this differential equation (iii) Apply Wronskin test to check that $y_1$ and $y_2$ are linearly independent. (5)

7. With the help of clear diagram define spherical and cylindrical polar coordinate systems. Derive the expressions for Laplacian in each of these two coordinate systems. (15)

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