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235103

December, 2019

BSc. (H) Physics - I SEMESTER

Calculus (OMTH-101)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Find the nth differential coefficient of $\log(ax + x^2)$. (1.5)

(b) Find the radius of curvature at the origin for

$$x^3 + 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0.$$ (1.5)

(c) Expand $\sin x$ in powers of $(x - \pi/2)$. (1.5)

(d) If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, then by using Euler's theorem, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$. (1.5)

(e) If $x = uv$, $y = \frac{u+v}{u-v}$, then, find $\frac{\partial(u,v)}{\partial(x,y)}$. (1.5)

(f) Evaluate $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2 \sin\theta \, dr \, d\theta$. (1.5)

(g) Evaluate $\iint_R xy(x+y)dxdy$ where R is the region bounded by $x^2 = y$ and $x = y$. (1.5)

(h) Find the surface area of a sphere of radius 'a'. (1.5)

(i) Evaluate $\int_0^1 x^4(1-\sqrt{x})^5 dx$ using Beta-Gamma function. (1.5)

(j) Evaluate $\iiint_R (x+y+z)dxdydz$, where R : $0 \le x \le 1$, $1 \le y \le 2$, $2 \le z \le 3$. (1.5)

PART - B

2. (a) Find the value of the nth derivative of $y = e^{m\sin^{-1}x}$ for $x = 0$ using Leibnitz's theorem. (8)

(b) Find all the asymptotes of the curve

$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$. (7)

3. (a) In a plane triangle ABC, find the maximum value of cos A cos B cos C. (8)

(b) Expand $(x^2y + \sin y + e^x)$ in powers of $(x - 1)$ and $(y - \pi)$ using Taylor's series for two variables. (7)

4. (a) The curve $y = 1/(1 + x^2)$ is rotated about x-axis between $x = -1$ and $x = 1$. Find the volume of the solid generated. (8)

(b) Change the order of integration $I = \int_0^1\int_{x^2}^{2-x} xydydx$ and hence evaluate the same. (7)

5. (a) Evaluate $\iiint xyzdxdydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. (8)

(b) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$. (7)

6. (a) If ρ is the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S is the focus of the parabola, then show that ρ^2 varies as $(SP)^3$. (8)

(b) Evaluate $\int_0^\infty \frac{e^{-x} \sin bx}{x} dx$ using differentiation under the integral sign. (7)

7. (a) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (8)

(b) State and prove the relation between Beta and Gamma functions. (7)
