Roll No. .... Total Pages : 3

335204

May 2019

B.Sc. (Physics/Chemistry) II Semester

LINEAR ALGEBRA

(OMTH-201)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Find k so that u and v are orthogonal, where $u = (1, k, -3)$ and $v = (2, -5, 4)$. (1.5)

(b) State Cauchy-Schwarz inequality and triangle inequality for vectors. (1.5)

(c) Define Rank of a matrix with example. (1.5)

(d) Write short note on vector space and subspace. (1.5)

(e) Determine whether u and v are linearly dependent where $u = 2t^2 + 4t - 3$, $v = 4t^2 + 8t - 6$. (1.5)

(f) Explain Basis and dimensions of a vector space. (1.5)

(g) Define transition matrix. (1.5)

(h) If $f : R^2 \to R^2$ is a mapping defined by $f(x, y) = (x + y, x)$ then show that 'f' is linear. (1.5)

(i) State Projection Theorem. (1.5)

(j) Define orthogonal projection onto a subspace. (1.5)

PART–B

2. (a) If $u = (1, -3, 4)$ and $v = (3, 4, 7)$ then find

(i) $\cos\theta$, where θ is the angle between u and v.

(ii) proj (u, v), projection of u onto v.

(iii) $d(u, v)$, the distance between u and v.

(iv) $\|u\|$ and $\|v\|$. (8)

(b) Find the eigen values and corresponding eigen vectors of the following matrix :

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$ (7)

3. (a) If $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ then find an orthogonal matrix P such that $D = P^{-1}AP$ is diagonal. (8)

(b) Show that the vectors $u = (1, 1, 1)$, $v = (1, 2, 3)$, $w = (1, 5, 8)$ span R^3. (7)

4. (a) Solve the following system of equations by Rank method

$x + 2y + z = 3$, $2x + 5y - z = -4$, $3x - 2y - z = 5$. (8)

(b) State and Prove Dimension theorem. (7)

5. If $f : R^4 \to R^3$ be a linear transformation defined by $f(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$
Then find a basis and the dimension of the

(i) Range of f

(ii) Kernel of f. (15)

6. (a) Reduce the following matrix to row canonical form

$$\begin{bmatrix} -4 & 1 & -6 \\ 1 & 2 & -5 \\ 6 & 3 & -4 \end{bmatrix}$$ (8)

(b) Determine whether or not the vectors $u = (1, 1, 2)$, $v = (2, 3, 1)$, $w = (4, 5, 5)$ in R^3 are linearly dependent. If they are linearly dependent, find a relation between them. (7)

7. (a) If a linear transformation $f : V \to U$ is one to one and onto then show that the inverse mapping $f^{-1} : U \to V$ is also linear.

(b) If W be the subspace of R^5 spanned by $u = (1, 2, 3, -1, 2)$ and $v = (2, 4, 7, 2, -1)$ then find a basis of the orthogonal complement $W^\perp$ of W. (8)