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May 2019

B.Sc. (Hons.) Physics Semester-IV

MATHEMATICAL PHYSICS-III

(BPH-401)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Find the modulus and principle argument of the complex number $\sqrt{\left(\frac{1+i}{1-i}\right)}$. (1.5)

(b) Separate $\log (x + iy)$ into real and imaginary parts. (1.5)

(c) State and prove Cauchy's Integral Theorem for the functions of complex variables. (1.5)

(d) Find the residue of $f(z) = \dfrac{1}{(z^2+1)^3}$ at $z = i$. (1.5)

(e) Find the poles of the function $f(z) = \dfrac{e^{z-a}}{(z-a)^2}$. (1.5)

(f) If F{$f(x)$} = F(s) then show that

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right).$$ (1.5)

(g) If F{$f(x)$} = F(s) then show that

$$F\left\{x^n f(x)\right\} = (-i)^n \frac{d^n}{ds^n} F(s).$$ (1.5)

(h) Evaluate the F.T. of Dirac-delta function. (1.5)

(i) If L{$f(t)$} = L(s) then show that

$$L\{f'(t)\} = s\,L(s) - f(0).$$ (1.5)

(j) Find the Laplace transform of $f(t) = \dfrac{\sin 2t}{t}$. (1.5)

PART–B

2. (a) Define a harmonic function. Show that the function $u(x, y) = x^4 - 6x^2y^2 + y^4$ is harmonic. Also find the analytic function $f(z) = u(x, y) + iv(x, y)$. (8)

(b) If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5 - 1 = 0$; find them and show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$. (7)

3. (a) Evaluate $\int_C \dfrac{12z-7}{(z-1)^2\ (2z+3)} dz$; using Cauchy's integral Formula where c is the circle $|z| = 3$. (7)

(b) Evaluate the following real integral using method of complex variables: (8)

$$\int_{-0}^{2\pi} \frac{\sin^2\theta}{5 - 4\cos\theta} d\theta.$$

4. (a) State and prove Taylor's theorem of complex variables. (8)

(b) Expand $f(z) = \dfrac{7z-2}{z^3 - z^2 - 2z}$ in a Laurent series in the region $|z+1| > 3$. (7)

5. (a) If F{$f(x)$} = F(s) then show that

$F\left\{f^n(x)\right\} = (-is)^n F(s)$ where $f^n(x)$ is the nth derivative of the function $f(x)$. (8)

(b) State and Prove convolution theorem on Fourier Transform. (7)

6. (a) Obtain Fourier Cosine transform of

$$F(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$ (8)

(b) Solve the following equation by Laplace Transform

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0;\ y = 0,\ dy/dx = 1 \text{ at } t = 0$$

and $y = 1$ at $t = \pi/8$. (7)

7. (a) Evaluate the Laplace transform of

$$F(t) = \begin{cases} 1, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ t^2, & 2 \le t < \infty \end{cases}$$ (8)

(b) Evaluate the integral $\int_0^\infty t^3 e^{-t} \sin t \, dt$ using Laplace transformation. (7)