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020202

May 2024

B. Tech. (RAI) (Second Semester)

Mathematics–II (BSC-106-RAI)

Time : 3 *Hours*] [*Maximum Marks :* 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Show that $f(z)=|z|^2$ is not analytic at any point. **1.5**

(b) Evaluate the integral $\int_C z^2\,dz$, where C is the arc of the circle $|z| = 2$ form $\theta = 0$ to $\theta = \pi/3$. **1.5**

(c) Find the constants a, b, c such that $f(z)=x-2ay+i(bx-cy)$ is analytic. **1.5**

(d) Compute the residues at the singular points of $f(z)=\frac{z}{(z+1)(z-2)}$. **1.5**

(e) Evaluate the double integral $\iint_R e^{x^2}\,dx\,dy$, where the region R is given by :

$R: 2y \le x \le 2$ and $0 \le y \le 1$. **1.5**

(f) Show that the vector :

$\vec{F}=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$

is irrotational. **1.5**

(g) State Bernoulli's equation. **1.5**

(h) Find the general solution of $y=(x-a)p-p^2$, where $p=\frac{dy}{dx}$. **1.5**

(i) Find the Wronskian of the functions xe^{-x} and e^{-x}. **1.5**

(j) Solve the differential equation :

$(D^2 - 4D + 3)\,y = 0$,

where $D \equiv \frac{d}{dx}$. **1.5**

Part B

2. (a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 4$, $z = 0$. 8

(b) Evaluate the following integration by changing the order of integration :

$\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\,dx\,dy$. 7

3. (a) Evaluate the integral :

$\int_C [(\sin x - y)\,dx - \cos x\,dy]$,

where C is the triangle with vertices (0, 0), $(\pi/2, 0)$ and $(\pi/2, 1)$. 8

(b) Solve the differential equation :

$2y\cos y^2 \frac{dy}{dx} - \frac{2}{x+1}\sin y^2 = (x+1)^3$. 7

4. (a) Solve the equation :

$xp^2 - 2yp + x = 0$,

where $p=\frac{dy}{dx}$. 8

(b) Show that the function :

$$u\ (x,\ y) = 2x + y^3 - 3x^2y$$

is harmonic. Find its harmonic conjugate $v\ (x,\ y)$ and corresponding analytic function $f(z)$. 7

5. (a) Use the residue theorem to evaluate the following integral :

$$\oint_C \frac{e^z - 1}{z(z-1)(z-i)^2}\,dz,$$

where (i) C : $|\ z\ | = 1/2$, (ii) C $|\ z\ | = 2$. 8

(b) Solve the differential equation :

$$(D^4 - 1)y = e^x \cos x,$$

where $D = \frac{d}{dx}$. 7

6. (a) Using the Cauchy integral theorem and its extension value $\oint_C \frac{dz}{z(z+2)}$, where C is any rectangle containing the points $z = 0$ and $z = -2$ inside it. 8

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} + y = \text{cosec}\, x \cot x$$

using method of variation of parameters. 7

7. Find all possible Taylor's and Laurent's series expansion of the function :

$$f(z) = 1/\left[(z+1)(z+2)^2\right]$$

about the point $z = 1$. 15