6. (a) Show that the function :

$$u = e^{-2xy}\sin\left(x^2 - y^2\right)$$

is Harmonic. Find the conjugate function v and express $u + iv$ as an analytic function of z. **8**

(b) Evaluate $\oint_C \frac{12z-7}{z(z-1)^2(2z+3)}dz$, where C is the circle : **7**

(i) $|z| = 3/2$

(ii) $|z| = 3$.

7. (a) Expand the function $\frac{7z-2}{z(z+1)(z-2)}$ in Taylor's series or Laurent's series expansion, when : **8**

(i) $0 < |z+1| < 1$

(ii) $1 < |z+1| < 3$.

(b) Evaluate by Contour Integration

$$\int_0^{2\pi} \frac{\sin^2\theta}{5-4\cos\theta}d\theta.$$ **7**

Roll No. ....

Total Pages : 04

008201

May 2024

B. Tech. (ECE/ENC/EEIOT) (Second Semester)

Mathematics–II (Calculus, Ordinary Differential Equations and Complex Variable (BSC-106D)

Time : 3 Hours] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) A vector field is given by :

$$F = \sin y\hat{i} + x(1+\cos y)\hat{j}.$$

Evaluate the line integral over a circular path given by $x^2 + y^2 = a^2, z = 0$. **1.5**

(b) Change the order of integration :

$$\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy.$$ **1.5**

(c) Solve $\left[1+\log(xy)\right]dx+\left[1+\frac{x}{y}\right]dy=0$. 1.5

(d) Solve $\left(D^2+5D-4\right)y=0$. 1.5

(e) Prove that $P_n(n)=1$. 1.5

(f) Prove that $\frac{d}{dx}J_0(x)=-J_1(x)$. 1.5

(g) Find p such that the function $f(z)=r^2\cos\theta+ir^2\sin p\theta$ is Analytic. 1.5

(h) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant. 1.5

(i) State Cauchy's Integral Formula. 1.5

(j) What is the region of w-plane into which the rectangular region is the z-plane bounded by the lines $x = 0$, $y = 0$, $x = 1$, $y = 2$ is mapped under the transformation $w = z + (2 - i)$. 1.5

Part B

2. (a) Evaluate $\iiint x^2yz\,dx\,dy\,dz$ throughout the volume bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. 8

(b) Evaluate $\iint_R (x+y)^2\,dx\,dy$, where R is a parallelogram in the xy-plane with vertices (1, 0), (3, 1), (2, 2), (0, 1) using the transformation $u=x+y, v=x-2y$. 7

3. (a) Verify Divergence Theorem for :

$$F=\left(x^2-yz\right)\hat{i}+\left(y^2-zx\right)\hat{j}+\left(z^2-xy\right)\hat{k}$$

taken over the rectangular parallelepiped $0\le x\le a, 0\le y\le b, 0\le z\le c$. 8

(b) Solve by Method of Variation of parameter $\left(D^2+1\right)y=\tan x$. 7

4. (a) Solve the Bernoulli's equation :

$$x^2dy+y(x+y)dx=0.$$ 8

(b) Find the Power series solution of $\left(1-x^2\right)y''-2xy'+2y=0$ about $x = 0$. 7

5. (a) Solve :

$$x^2\frac{d^2y}{\partial x^2}+x\frac{dy}{dx}+y=\sin\left(\log x^2\right).$$ 8

(b) Solve :

$$\left(D^2-4D+4\right)y=8x^2e^{2x}\sin 2x.$$ 7