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002202

May 2024

B.Tech. (CIVIL/ENV) - II SEMESTER

Mathematics-II (Differential Equations) (BSC-106B)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Determine the order and degree of the given differential equation : (1.5)

$$\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)^{-3/2} = 0.$$

(b) Solve $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$. (1.5)

(c) Find the value of k for which the differential equation (1.5)

$(xy^2 + kx^2y)dx + (x + y)x^2dy = 0$ is exact.

(d) Find the solution of the differential equation

$$\frac{d^2y}{dx^2}+y=0.$$ (1.5)

(e) Solve $zp = -x$, where symbols have their usual meanings. (1.5)

(f) Solve the following partial differential equation $p^2 - q^2 = 1$. (1.5)

(g) Classify the following partial differential equation : (1.5)

$$2(\partial^2 u/dx^2) + 4(\partial^2 u/dxdy) + 3(\partial^2 u/dy^2) = 2.$$

(h) Solve $(D - 3D' - 2)^2 z = 0$. (1.5)

(i) Solve $(D - D'^2)z = 0$. (1.5)

(j) Solve $p = \sin(y - xp)$, where symbols have their usual meanings. (1.5)

PART–B

2. (a) Solve $x(3ydx + 2xdy) + 8y^4(ydx + 3xdy) = 0$. (8)

(b) Solve the following differential equation : (7)

$$p^2 + 2py \cot x = y^2.$$

3. (a) Solve the following differential equation : (7)

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y.$$

(b) Solve (8)

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x.\sin(\log x).$$

4. (a) Solve the following differential equation : (5)

$$x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0.$$

(b) Solve the equation $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ in series. (10)

5. (a) Solve $xyp + y^2q = zxy - 2x^2$. (7)

(b) Find a complete integral of $z^2(p^2 + q^2) = x^2 + y^2$. (8)

6. (a) Determine u such that $\partial^2u/\partial x^2 = (1/k).(\partial u/\partial t)$ and satisfy the conditions (5)

(i) $u \to 0$ as $t \to \infty$ (ii) $u = \sum_n C_n \cos nx$, for $t = 0$.

(b) Solve $(x^2D^2 - 2xyDD' - 3y^2D'^2 + xD - 3yD')\, z = x^2y \cos(\log x^2)$. (10)

7. Show that the general solution of wave equation $c^2(\partial^2u/\partial x^2) = \partial^2u/\partial t^2$ is $u(x, t) = \phi(x + ct) + \psi(x - ct)$, where ϕ and ψ are arbitrary functions. (15)