Roll No. ....   Total Pages : 05

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May 2024

B.Tech. (EL) (Second Semester)

Mathematics (Linear Algebra, Transform Calculus and Numerical Methods) (BSC-106C)

Time : 3 Hours]   [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Find the eigen values of the matrix : **1.5**

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$

(b) Explain Elementary Row transformations with the help of examples. **1.5**

(c) Find the rank of the matrix : **1.5**

$$\begin{bmatrix} 1 & 3 & 4 & -1 \\ 2 & -2 & 6 & 3 \end{bmatrix}.$$

(d) The function $f(x)$ is given by : **1.5**

x	0	0.5	1
$f(x)$	1	0.8	0.5

Then using Trapezoidal rule, find the value of $\int_0^1 f(x)dx$.

(e) Derive the iterative formula for finding $\frac{1}{\sqrt{N}}$ using Newton's Raphson method, where N is a real number. **1.5**

(f) Write down the formula for Adams-Bashforth method for finding the solution of the problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. **1.5**

(g) Write down the Schmidt explicit formula for finding the solution of the equation $u_t = c^2 u_{xx}$ **1.5**

(h) Find the laplace transform of : **1.5**

$$f(t) = e^{2t} + 4t^3 - 2\sin 3t.$$

(i) Find the inverse Laplace transform of $\frac{4s+3}{s^2+4}$. **1.5**

(j) State Convolution theorem for Laplace Transform. **1.5**

Part B

2. (a) Test for consistency and solve the following system of equations : 7

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$

(b) Verify Cayley-Hamilton theorem for the matrix A :

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & -2 \end{bmatrix}$$

and hence find A^{-1}. 8

3. (a) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of Regula-Falsi correct up to three decimal places. 8

(b) From the given table, compute the value of sin 38° using Newton's Backward Interpolation formula : 7

x^0	sin x
0	0
10	0.17365
20	0.34202
30	0.5
40	0.64279

4\. (a) Using Runge-Kutta method of order four to find the approximate value of y for $x = 0.2$, in steps of 0.1 if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ where $x = 0$. 8

(b) Evaluate the following integral by using the Simpson's 1/3rd rule. $\int_0^{0.6} e^{-x^2} dx$. (Take $h = 0.2$) 7

5\. (a) Solve the differential equation using Laplace transform 8

$y'' - 3y' + 2y = 4t + e^{3t}$, when $y(0) = 1$ and $y'(0) = -1$.

(b) Solve the following integral using Laplace transform : 7

$$\int_0^{\infty} te^{-3t} \sin t \, dt.$$

6\. (a) Diagonalise the following matrix : 7

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b) Using Milne's predictor-corrector method to find the value of $y(4.5)$ given that $5x\frac{dy}{dx} + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$, $= y(4.4) = 1.0187$. 8

7\. (a) Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = y = 0$, $x = y = 3$ with $u = 0$ on the boundary and mesh length = 1. 8

(b) Apply Convolution Theorem to evaluate the $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$. 7