Sr. No. 002406

May 2024

B.Tech. (Civil/ENV) 4th Sem

Mathemetics-III (BSC-201)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Six cards are drawn at random from a pack of 52 cards. What is the probability that 3 will be red and 3 black? (1.5)

(b) What is a random variable? Also, discuss its different types. (1.5)

(c) A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find the value of k. (1.5)

(d) Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean of the number of kings. (1.5)

(e) Form the partial differential equation by eliminating h and k from the equation $(x-h)^2+(y-k)^2+z^2=\beta^2$. (1.5)

(f) Solve $yzp+zxq=xy$. (1.5)

(g) Classify the p.d.e. $\frac{\partial^2 z}{\partial x^2}-5\frac{\partial^2 z}{\partial x\partial y}+6\frac{\partial^2 z}{\partial y^2}=0$. (1.5)

(h) Write one-dimensional wave and heat equation. (1.5)

(i) If the sum of the mean and the variance of the binomial distribution of 5 trials is 4.8, find the distribution. (1.5)

(j) What is correlation and its different types? (1.5)

PART -B

Q2 (a) Solve $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y} - x^2y$. (8)

(b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ subject to the following boundary and initial conditions

$u(0,t) = u(a,t) = 0,\ t > 0;\ \left[\frac{\partial u}{\partial t}\right]_{t=0} = 0, 0 < x < a$ and $u(x,0) = k(ax - x^2), 0 < x < a, k$ being a suitable constant. (7)

Q3 (a) In a lot of 500 solenoids 25 are defective. Find the probability of 0,1,2,3, defective solenoids in a random sample of 20 solenoids. (8)

(b) A discrete variable X can assume the values $x = 1,2,3, \dots \dots \dots$ with probabilities 2^{-x}. Show that Chebyshev's inequality gives $P(|X-2| \leq 2) > \frac{1}{2}$, while the actual probability is 15/16. (7)

Q4 (a) Random sample of 400 men and 600 women were asked whether they would to have a school near their residence. 200 men and 325 women were in favor of the proposal. Test the hypothesis that the proportion of men and women in favor of the proposal are same at 5% level of significance. (Given z value at 5% level is 1.96) (8)

(b) From the following table, calculate the rank correlation coefficient:

X	48	33	40	9	16	16	65	24	16	57
Y	13	13	24	6	15	4	20	9	6	19

(7)

Q5 (a) The customer accounts of certain department store have an average balance of Rs 120 and a standard deviation of Rs 40. Assuming that the account balances are normally distributed:

(i) What proportion of the account is over Rs 150?

(ii) What proportion of the account is between Rs 100 and Rs 150?

(iii) What proportion of the account is between Rs 60 and Rs 90?

[Given

$P(z > 0.75) = 0.2266, P(-0.5 < z > 0.75) = 0.4649, P(-1.5 < z > -0.75) = 0.1598$] (8)

(b) Records taken of the number of male and female births in 800 families having four children are as follows:

No. of male births	0	1	2	3	4
No. of female births	4	3	2	1	0

No. of families	32	178	290	236	94

Test whether the data are consistent with the hypothesis that the binomial law holds and the chance of male birth is equal to that of female birth, namely

$p = q = 1/2$. (Table value of χ^2 at 5% level of significance for 4 degree of freedom is 9.49).

(7)

Q6 (a) Fit a second degree curve of regression of y on x to the following data:

x	1	2	3	4
y	6	11	18	27

(8)

(b) Three fair coins are tossed. Let X denote the number of heads on the first two coins, and let Y denote the numbers of tails on the last two coins. Then

(i) Find the joint distribution of X and Y.

(ii) Find the conditional distribution of Y given that $X = 1$.

(iii) Find $Cov\ (X, Y)$ (7)

Q7 (a) Solve $z(xy + z^2)(px - qy) = x^4$. (8)

(b) Solve the following wave equation by using D'Alembert's Method

$u_{tt} = u_{xx},\ u(x,0) = \sin x,\ u_t(x,0) = 1$, then find $u(x,t)$ and $u\left(\pi, \frac{\pi}{2}\right)$. (7)