Sr. No 753402

Dec 2023

M.Sc (Mathematics) IV SEMESTER

Differential Geometry(MATH21-852) Supplementary Exam

Time: 3 Hours

Max. Marks:75

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART –A

Q1 (a) Find the arc length of the curve given as intersection of the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, x = a\cosh\left(\frac{z}{a}\right)$ from the point (a,0,0) to the point (x,y,z). (1.5)

(b) Show that the principal normals at consecutive points do not intersect unless $\tau = 0$. (1.5)

(c) Show that the curves $du^2 - (u^2 + c^2)(d\emptyset)^2 = 0$ form an orthogonal system on the surface $x = u\cos\emptyset, y = u\sin\emptyset, z = c\emptyset$ (1.5)

(d) If K, τ are the curvature and Torsion of a geodesic , Prove that (1.5)

$$\tau^2 = (K - K_a)(K_b - K)$$

(e) Find envelope of the sphere $(x - a\cos\theta)^2 + (y - a\sin\theta)^2 + z^2 = b^2$. (1.5)

(f) Show that $[\vec{n}, \vec{n_1}, \vec{r_2}] = \frac{EM-GL}{H}$ (1.5)

(g) Prove that the surface $xy = (z - c)^2$ is developable. (1.5)

(h) Prove that the tangent plane to the surface $xyz = a^3$ and the coordinate planes bound a tetrahedron of constant volume . (1.5)

(i) Show that if L,M,N vanish everywhere on a surface then the surface is part of a plane. (1.5)

(j) Define Gaussian Curvature. (1.5)

PART –B

Q2 (a) For the curve $x = 4a\cos^3 u, y = 4a\sin^3 u, z = 3c\cos 2u$ prove that (8)

$k = \frac{a}{6(a^2+c^2)\sin 2u}$. find its Torsion also.

(b) If s_1 is the arc length of locus of the curve of curvature , show that (7)

$$\frac{ds_1}{ds} = \frac{\sqrt{(\kappa^2\tau^2+\kappa'^2)}}{\kappa^2} = \sqrt{[(\frac{\rho}{\sigma})^2 + \rho'^2]}$$

Q3 (a) Show that the sum of squares of the intercepts on the co-ordinate axes made by the tangent planes to the surface $x^{2/3} + y^{2/3} = a^{2/3}$ is constant. (8)

(b) Find the edge of regression of the envelope the family of planes (7)

$$x\sin\theta - y\cos\theta + y = a\theta.$$

Q4 (a) Find the Condition that the two directions given by the quadratic $Pdu^2 + Qdu\,dv + Rdv^2 = 0$ are orthogonal . (8)

(b) On a paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z=constant.. (7)

Q5 (a) Find the direction coefficients of the direction making an angle $\frac{\pi}{2}$ with the direction having direction coefficients (l,m). (8)

(b) If the tangent and binormal at a point of a curve make angles $\theta, \emptyset$ respectively with a fixed direction , show that (7)

$$\frac{\sin\theta\, d\theta}{\sin\emptyset\, d\emptyset} = \frac{-K}{\tau}.$$

Q6 (a) Prove that (8)

$$T^2\overrightarrow{r_1} = (FM - EN)\overrightarrow{n_1} + (EM - FL)\overrightarrow{n_2}$$
$$T^2\overrightarrow{r_2} = (GM - FN)\overrightarrow{n_1} + (FM - GL)\overrightarrow{n_2}$$

(b) Calculate the fundamental magnitudes of the surface (7)

$$x = a(u+v), y = b(u-v), z = uv.$$

Q7 (a) Derive the Mainardi-Codazzi equation . (8)

(b) Prove that the Torsion of an asymptotic line is equal to the torsion of its Geodesic tangent. (7)